2021 Croucher Summer Course in Information Theory

# Fair machine learning

## Lecture 2

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# A fair classifier using kernel density estimation

Reading: TN2

## **Recap: MI-based optimization**

$$\min_{w} \frac{1-\lambda}{m} \sum_{i=1}^{m} \ell_{\mathsf{CE}}(y^{(i)}, \hat{y}^{(i)}) + \lambda \cdot I(Z; \hat{Y})$$

**Mentioned:** training instability.

**Rationale** behind training instability:

$$I(Z;\hat{Y})$$
 = "max" optimization

"min-max" optimization often suffers from training instability.

Recap

$$\min_{w} \frac{1-\lambda}{m} \sum_{i=1}^{m} \ell_{\mathsf{CE}}(y^{(i)}, \hat{y}^{(i)}) + \lambda \cdot I(Z; \hat{Y})$$

**Claimed:** There is another fair classifier that addresses training instability while offering a better tradeoff.

Will study the new fair classifier in depth.

- 1. Explore a way to directly compute the fairness measure DDP.
- 2. Introduce a trick that allows us to well approximate DDP: Kernel Density Estimation (KDE)
- 3. Develop a KDE-based optimization for a fair classifier.
- 4. Study how to solve the optimization.

## **Revisit: the fairness measure DDP**

$$\mathsf{DDP} := \sum_{z \in \mathcal{Z}} |\mathbb{P}(\tilde{Y} = 1 | Z = z) - \mathbb{P}(\tilde{Y} = 1)|$$

Let's try to compute this directly.

First focus on:

$$\begin{split} \mathbb{P}(\tilde{Y} = 1) &= \mathbb{P}(\hat{Y} \geq \tau) & \tilde{Y} := \mathbf{1}\{\hat{Y} \geq \tau\} \\ &= \int_{\tau}^{\infty} \underbrace{f_{\hat{Y}}(t)dt}_{\mathbf{pdf uknown!}} \end{split}$$

**Instead:** We are given samples  $\{\hat{y}^{(1)}, \dots, \hat{y}^{(m)}\}$ **Question:** A way to infer the pdf from samples?

## Kernel density estimation (KDE)

$$\mathbb{P}(\tilde{Y}=1) = \int_{\tau}^{\infty} f_{\hat{Y}}(t) dt$$

Given samples  $\{\hat{y}^{(1)}, \dots, \hat{y}^{(m)}\}$ , KDE is defined as:  $\widehat{f}_{\hat{Y}}(t) := \frac{1}{mh} \sum_{i=1}^{m} f_{ker} \left( \frac{t - \hat{y}^{(i)}}{h} \right)$ a smoothing parameter (bandwidth) (e.g., Gaussian kernel)

## Accuracy of KDE?

$$\mathbb{P}(\tilde{Y}=1) = \int_{\tau}^{\infty} f_{\hat{Y}}(t) dt$$

Given samples 
$$\{\hat{y}^{(1)}, \dots, \hat{y}^{(m)}\}$$
, KDE is defined as:  
 $\hat{f}_{\hat{Y}}(t) := \frac{1}{mh} \sum_{i=1}^{m} f_{ker} \left(\frac{t - \hat{y}^{(i)}}{h}\right)$ 

Jiang ICML17:  $|\widehat{f}(t) - f(t)|_{\infty} \lesssim \frac{1}{m^{\frac{1}{d}}}$  dim. of an interested r.v.

→ Yields an inaccurate estimate under high-dim. cases Good news: In our setting, d = 1

## **Approximation via KDE**

$$\begin{split} \mathbb{P}(\tilde{Y} = 1) &= \int_{\tau}^{\infty} f_{\hat{Y}}(t) dt \\ \widehat{\mathbb{P}}(\tilde{Y} = 1) &= \int_{\tau}^{\infty} \widehat{f}_{\hat{Y}}(t) dt \\ &= \int_{\tau}^{\infty} \frac{1}{mh} \sum_{i=1}^{m} f_{\text{ker}}\left(\underbrace{\frac{t - \hat{y}^{(i)}}{h}}_{h}\right) dt \\ &= \frac{1}{m} \sum_{i=1}^{m} \int_{\frac{\tau - \hat{y}^{(i)}}{h}}^{\infty} f_{\text{ker}}(y) dy \\ &= \frac{1}{m} \sum_{i=1}^{m} Q\left(\frac{\tau - \hat{y}^{(i)}}{h}\right) \text{ (Gaussian kernel)} \end{split}$$

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## **Approximation via KDE**

$$\widehat{\mathbb{P}}(\widetilde{Y}=1) = \frac{1}{m} \sum_{i=1}^{m} Q\left(\frac{\tau - \widehat{y}^{(i)}}{h}\right)$$

Remember: DDP :=  $\sum_{z \in \mathcal{Z}} |\mathbb{P}(\tilde{Y} = 1 | Z = z) - \mathbb{P}(\tilde{Y} = 1)|$ 

Similarly, one can obtain:

$$\widehat{\mathbb{P}}(\widetilde{Y} = 1 | Z = z) = \frac{1}{m_z} \sum_{i \in I_z} Q\left(\frac{\tau - \widehat{y}^{(i)}}{h}\right)$$
$$|I_z| \qquad \{i : z^{(i)} = z\}$$

## **Approximated DDP**



Can express DDP in terms of samples (thus *w*)



**Issues**: How to deal with the absolute function? How to choose bandwidth *h*?

## How to deal with the absolution func?

$$\min_{w} \frac{1-\lambda}{m} \sum_{i=1}^{m} \ell_{\mathsf{CE}}(y^{(i)}, \hat{y}^{(i)}) + \frac{\lambda}{m} \cdot \sum_{z \in \mathcal{Z}} \left| \frac{m}{m_z} \sum_{i \in I_z} e^{-\frac{(\tau - \hat{y}^{(i)})^2}{2h^2}} - \sum_{i=1}^{m} e^{-\frac{(\tau - \hat{y}^{(i)})^2}{2h^2}} \right|$$

Instead, one can employ Huber loss:

$$H_{\delta}(x) = \frac{1}{2}x^{2} \qquad \text{if } |x| \leq \delta$$

$$\begin{cases} \delta\left(|x| - \frac{1}{2}\delta\right) & \text{otherwise} \end{cases}$$

This enables us to readily obtain gradient.

## How to choose bandwidth h?

$$\min_{w} \frac{1-\lambda}{m} \sum_{i=1}^{m} \ell_{\mathsf{CE}}(y^{(i)}, \hat{y}^{(i)}) + \frac{\lambda}{m} \cdot \sum_{z \in \mathcal{Z}} H_{\delta}\left(\frac{m}{m_{z}} \sum_{i \in I_{z}} e^{-\frac{(\tau - \hat{y}^{(i)})^{2}}{2h^{2}}} - \sum_{i=1}^{m} e^{-\frac{(\tau - \hat{y}^{(i)})^{2}}{2h^{2}}}\right)$$

#### Turns out:

There is a sweet spot for h that miminizes the mean square error of KDE estimate.

Advise us to find  $h^*$  that minimizes the MSE.

See [Cho-Hwang-Suh NeurIPS20] for details.

#### Extension to another fairness measure **DEO**

$$\mathsf{DEO} := \sum_{y \in \mathcal{Y}} \sum_{z \in \mathcal{Z}} |\mathbb{P}(\tilde{Y} = 1 | \mathbf{Y} = \mathbf{y}, Z = z) - \mathbb{P}(\tilde{Y} = 1 | \mathbf{Y} = \mathbf{y})|$$

$$\approx \sum_{y \in \mathcal{Y}} \sum_{z \in \mathcal{Z}} |\widehat{\mathbb{P}}(\widetilde{Y} = 1 | Y = y, Z = z) - \widehat{\mathbb{P}}(\widetilde{Y} = 1 | Y = y)$$
  
$$\approx \sum_{y \in \mathcal{Y}} \sum_{z \in \mathcal{Z}} \left| \frac{1}{m_{yz}} \sum_{i \in I_{yz}} e^{-\frac{(\tau - \hat{y}^{(i)})^2}{2h^2}} - \frac{1}{m_y} \sum_{i \in I_y} e^{-\frac{(\tau - \hat{y}^{(i)})^2}{2h^2}} \right|$$
  
$$|I_{yz}| \qquad \{i : y^{(i)} = y, z^{(i)} = z\}$$

## **Experiments**

#### A benmark real dataset: **COMPAS**



(x, z, y)

criminal records

black or white reoffend or not

## **Trainining instability?**



## Accuracy vs DDP tradeoff

	Accuracy	DDP
Non-fair classifier	$68.29 \pm 0.44$	$0.2263 \pm 0.0087$
MI-based fair classifier	$67.07 \pm 0.85$	$0.0522 \pm 0.0373$
KDE-based fair classifier	$67.00 \pm 0.45$	$0.0374 \pm 0.0079$

## Accuracy vs DDP tradeoff



- 1. Explore fairness measures in fair classifiers.
- 2. Study an MI-based fair classifier which yields a good tradeoff while suffering from training instability.
- 3. Investigate another fair classifer based on KDE, which addresses the training instability issue.

## **Revisit: Five aspects for trustworthy AI**

A recent progress: Roh-Lee-Whang-Suh, ICML20









explainability

value alignment

transparency

# Will explore the recent work on fairness & robustness, and discuss relative issues.

### Reference

[1] J. Cho, G. Hwang and C. Suh. A fair classifier using mutual information. *IEEE International Syposium on Inofrmation Theory (ISIT)*, 2020.

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[4] J. Angwin, J. Larson, S. Mattu, and L. Kirchner. Machine bias: There's software used across the country to 272 predict future criminals. And it's biased against blacks. *https://www.propublica.org/article/machine-bias-risk-assessments-incriminal-sentencing*, 2015.