Two-Way Interference Channel Capacity: How to Have the Cake and Eat it Too

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***Abstract*—Two-way communication is prevalent and its fundamental limits are first studied in the point-to-point setting by Shannon [1]. One natural extension is a two-way interference channel (IC) with four independent messages: two associated with each direction of communication. In this work, we explore a deterministic two-way IC which captures key properties of the wireless Gaussian channel. Our main contribution lies in the complete capacity region characterization of the two-way IC (w.r.t. the forward and backward sum-rate pair) via a new achievable scheme. One surprising consequence of this result is that not only we can get an interaction gain over the one-way non-feedback capacities, we can sometimes *get all the way to perfect feedback capacities in both directions simultaneously*.**

I. Introduction

Two-way communication, where two nodes want to communicate data to each other, is prevalent. The first study of such two-way channels was done by Shannon [1] in the setting of point-to-point memoryless channels. When the point-to-point channels in the two directions are orthogonal (such as when the two directions are allocated different time slots or different frequency bands, or when the transmitted signal can be canceled perfectly as in full-duplex communication), the problem is not interesting as feedback does not increase point-to-point capacity. Hence, communication in one direction cannot increase the capacity of the other direction and no *interaction gain* is possible. One can achieve no more than the one-way capacity in each direction.

The situation changes in network scenarios where feedback can increase capacity. In these scenarios, communication in one direction can potentially increase the capacity of the other direction by providing feedback in addition to communicating data. One scenario of particular interest is the setting of the two-way interference channel (two-way IC), modeling two interfering two-way communication links (Fig. 1). Not only is this scenario common in wireless communication networks, it has also been demonstrated that feedback provides a significant gain for communication over (one-way) IC's [2], [3], [4]. In particular, [3] reveals that the feedback gain can be unbounded, i.e., the gap between the feedback and non-feedback capacities can be arbitrarily large for certain channel parameters. This suggests the potential of significant interaction gain in two-way IC's. On the other hand,

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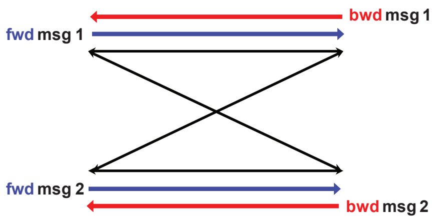

Fig. 1. Two interfering two-way communication links, consisting of two IC's, one in each direction. The IC's are orthogonal to each other and do not necessarily have the same channel gains.

the feedback result [3] assumes a dedicated infinite-capacity feedback link. In the two-way setting, any feedback needs to be transmitted through a backward IC, which also needs to carry its own backward data traffic. The question is when we take in consideration the competition with the backward traffic, whether there is still any net interaction gain through feedback?

To answer this question, [5] investigated a two-way IC under the linear deterministic model [6], which approximates a Gaussian channel. A scheme is proposed to demonstrate a net interaction gain, i.e., one can simultaneously achieve better than the non-feedback capacities in both directions. While an outer bound is also derived, it has a gap to the lower bound. Hence, there has been limited understanding on the maximal gain that can be reaped by feedback. In particular, whether or not one can get all the way to perfect feedback capacities in both directions has been unanswered.

Recent efforts have been made towards outer bounds. Cheng-Devroye [7] derived an outer bound, but it does not give a proper answer as the result assumes a partial interaction scenario in which interaction is enabled only at two nodes, while no interaction is permitted at the other two nodes. Under the full interaction scenario of interest, a new outer bound is established in [8]. However, the improved bound still comes with a gap to the lower bound and thus the maximal feedback gain has been unknown so far.

In this work, we settle this open problem and completely characterize the capacity region of the deterministic two-way IC via a new capacity-achieving transmission scheme. For simplicity, we assume the IC in each direction is symmetrical between the two users; however the IC's in the two directions are not necessarily the same (for example, they may use different frequency bands). For some channel gains, the new

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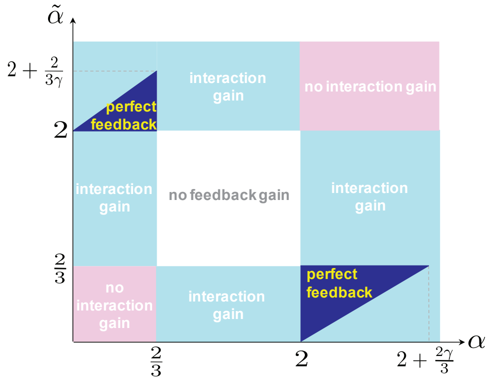

Fig. 2. **When can one have the cake and eat it too?** The plot is over two channel parameters of the deterministic model, α and $\tilde{\alpha}$, where α is the ratio of the interference-to-noise ratio (in dB) to the signal-to-noise ratio (in dB) of the IC in the forward direction and $\tilde{\alpha}$ is the corresponding quantity of the IC in the backward direction. The parameter γ is the ratio of the backward signal-to-noise ratio (in dB) to the forward signal-to-noise ratio (in dB), and is fixed to be a value between 1 and 4. White region: feedback does not increase capacity in either direction and thus interaction is not useful. Purple: feedback does increase capacity but interaction cannot provide such increase. Light blue: feedback can be provided through interaction and there is a net interaction gain. Dark blue: interaction is so efficient that one can achieve perfect feedback capacity simultaneously in both directions. This implies that one can obtain the maximal feedback gain without any sacrifice for feedback transmission (have the cake and eat it too).

scheme simultaneously achieves the perfect feedback sum-capacities of the IC's in both directions. This occurs even when feedback offers gains in both directions and thus feedback must be explicitly or implicitly carried over each IC while sending the traffic in its own direction. Fig. 2 shows for what channel gains this happens.

In the new scheme, feedback allows the exploitation of the following as side information: (i) past received signals; (ii) users' own messages; (iii) even the *future* information via *retrospective decoding* (to be detailed later; see Remark 2 in particular). While the first two were already shown to offer a feedback gain in literature, the third is newly exploited. It turns out this new exploitation leads us to achieve the perfect feedback capacities in both directions, which can never be done by the prior schemes [3], [4], [5].

II. Model

Fig. 3 describes a two-way deterministic IC where user k wants to send its own message W_k to user $\tilde{k}$, while user $\tilde{k}$ wishes to send its own message $\tilde{W}_k$ to user k, $k = 1, 2$. We assume that $(W_1, W_2, \tilde{W}_1, \tilde{W}_2)$ are independent and uniformly distributed. For simplicity, we consider a setting where both forward and backward ICs are symmetric but not necessarily the same. In the forward IC, n and m indicate the number of signal bit levels for direct and cross links respectively. The corresponding values in the backward IC are denoted by $(\tilde{n}, \tilde{m})$. Let $X_k \in \mathbb{F}_2^{\max(n,m)}$ be user k's transmitted signal and $V_k \in \mathbb{F}_2^m$ be a part of X_k visible to user $\tilde{j}(\neq \tilde{k})$. Similarly let $\tilde{X}_k$ be user $\tilde{k}$'s transmitted signal and $\tilde{V}_k$ be a part of $\tilde{X}_k$ visible to user $j(\neq k)$. The deterministic model abstracts broadcast and superposition of signals in the wireless Gaussian channel. See [6] for explicit details. A signal bit

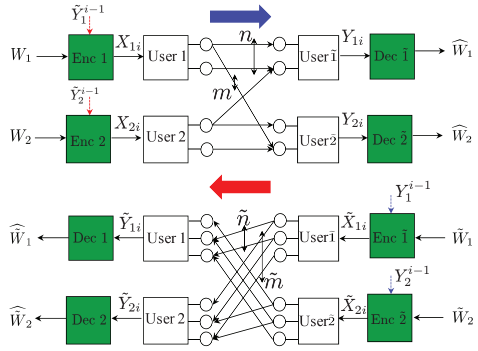

Fig. 3. A two-way deterministic interference channel (IC).

level observed by both users is broadcasted. If multiple signal levels arrive at the same signal level at a user, we assume modulo-2-addition. The encoded signal X_{ki} of user k at time i is a function of its own message and past received signals: $X_{ki} = f_{ki}(W_k, \tilde{Y}_k^{i-1})$. We define $\tilde{Y}_k^{i-1} := \{\tilde{Y}_{kt}\}_{t=1}^{i-1}$ where $\tilde{Y}_{kt}$ denotes user k's received signal at time t, offered through the backward IC. Similarly the encoded signal $\tilde{X}_{ki}$ of user $\tilde{k}$ at time i is a function of its own message and past received signals: $\tilde{X}_{ki} = \tilde{f}_{ki}(\tilde{W}_k, Y_k^{i-1})$.

A rate tuple $(R_1, R_2, \tilde{R}_1, \tilde{R}_2)$ is said to be achievable if there exists a family of codebooks and encoder/decoder functions such that the decoding error probabilities go to zero as code length N tends to infinity. Since the fundamental tradeoff w.r.t. (R_1, R_2) (or $(\tilde{R}_1, \tilde{R}_2)$) has already been characterized in [3], we focus on a sum-rate pair regarding the forward and backward ICs: $(R, \tilde{R}) := (R_1 + R_2, \tilde{R}_1 + \tilde{R}_2)$[1]. The capacity region is defined as the closure of the set of achievable sum-rate pairs: $\mathcal{C} = \text{closure}\{(R, \tilde{R}) : (R_1, R_2, \tilde{R}_1, \tilde{R}_2) \in \mathcal{C}_{\text{high}}\}$ where $\mathcal{C}_{\text{high}}$ denotes the one w.r.t. the high-dimensional tuple.

III. Main Results

Theorem 1 (Capacity region): The capacity region $\mathcal{C}$ of the two-way IC is the set of $(R, \tilde{R})$ such that

$$R \le \max(2n - m, m) =: C_{\text{pf}} \quad (1)$$
$$\tilde{R} \le \max(2\tilde{n} - \tilde{m}, \tilde{m}) =: \tilde{C}_{\text{pf}} \quad (2)$$
$$R + \tilde{R} \le 2(n + \tilde{n}) \quad (3)$$
$$R + \tilde{R} \le 2\max(n - m, m) + 2\max(\tilde{n} - \tilde{m}, \tilde{m}) \quad (4)$$

where C_{pf} and $\tilde{C}_{\text{pf}}$ indicate the perfect feedback sum-capacities of the forward and backward IC's, respectively [3].

Proof: See Section IV for achievability. For converse, we first note that the first two bounds match the perfect feedback bound [3], [9], [5]. The third bound is due to cutset: $R_1 + \tilde{R}_2 \le n + \tilde{n}$, $R_2 + \tilde{R}_1 \le n + \tilde{n}$. The last bound is proved in [8]. ■

We state two baselines for comparison to our main result.

Baseline 1 ([10], [11]): The capacity region $\mathcal{C}_{\text{no}}$ for the non-interactive scenario is the set of $(R, \tilde{R})$ such that

$$R \le \min\{2\max(n - m, m), \max(2n - m, m), 2n\} =: C_{\text{no}}$$
$$\tilde{R} \le \min\{2\max(\tilde{n} - \tilde{m}, \tilde{m}), \max(2\tilde{n} - \tilde{m}, \tilde{m}), 2\tilde{n}\} =: \tilde{C}_{\text{no}}.$$

[1]The extension to the four-rate tuple case is not that challenging although it requires a complicated analysis. Since the case does not provide any additional insights, here we consider a simpler sum-rate pair setting.

Baseline 2 ([3]): The capacity region for the perfect feedback scenario is $\mathcal{C}_{\sf pf} = \{(R, \tilde{R}) : R \le C_{\sf pf}, \tilde{R} \le \tilde{C}_{\sf pf}\}$.

With Theorem 1 and Baseline 1, one can readily see that feedback gain (in terms of capacity region) occurs as long as $(\alpha \notin [\frac{2}{3}, 2], \tilde{\alpha} \notin [\frac{2}{3}, 2])$, where $\alpha := \frac{m}{n}$ and $\tilde{\alpha} := \frac{\tilde{m}}{\tilde{n}}$. A careful inspection reveals that there are channel regimes in which one can enhance $C_{\sf no}$ (or $\tilde{C}_{\sf no}$) without sacrificing the other counterpart. This implies a net interaction gain.

Definition 1 (Interaction gain): We say that an interaction gain occurs if one can achieve $(R, \tilde{R}) = (C_{\sf no} + \delta, \tilde{C}_{\sf no} + \tilde{\delta})$ for some $\delta \ge 0$ and $\tilde{\delta} \ge 0$ such that $\max(\delta, \tilde{\delta}) > 0$.

A tedious yet straightforward calculation with this definition leads us to identify channel regimes which exhibit an interaction gain, marked in light blue in Fig. 2.

Achieving perfect feedback capacities: One interesting observation is that there are channel regimes in which both δ and $\tilde{\delta}$ can be strictly positive. This is unexpected because it implies that not only feedback does not sacrifice one transmission for the other, it can actually improve both simultaneously. More interestingly, δ and $\tilde{\delta}$ can reach up to the maximal feedback gains, reflected in $C_{\sf pf} - C_{\sf no}$ and $\tilde{C}_{\sf pf} - \tilde{C}_{\sf no}$. The dark blue regimes in Fig. 2 indicate such channel regimes when $1 \le \gamma := \frac{\tilde{n}}{n} \le 4$. Note that such regimes depend on γ. The amount of feedback that one can send is limited by available resources offered by the backward (or forward) IC. Hence, the feedback gain can be saturated depending on availability of the resources, which is affected by the channel asymmetry parameter γ. One point to note here is that for any γ, there always exists a non-empty set of $(\alpha, \tilde{\alpha})$ in which perfect feedback capacities can be achieved. Corollary 1 stated below exhibits all of such channel regimes.

Corollary 1: Consider a case in which feedback helps in both ICs: $C_{\sf pf} > C_{\sf no}$ and $\tilde{C}_{\sf pf} > \tilde{C}_{\sf no}$. In this case, the channel regimes in which $\mathcal{C} = \mathcal{C}_{\sf pf}$ are: (I) $\alpha < 2/3, \tilde{\alpha} > 2, C_{\sf pf} - C_{\sf no} \le 2\tilde{m} - \tilde{C}_{\sf pf}$, $\tilde{C}_{\sf pf} - \tilde{C}_{\sf no} \le 2n - C_{\sf pf}$; (II) $\tilde{\alpha} < 2/3, \alpha > 2$, $\tilde{C}_{\sf pf} - \tilde{C}_{\sf no} \le 2m - C_{\sf pf}$, $C_{\sf pf} - C_{\sf no} \le 2\tilde{n} - \tilde{C}_{\sf pf}$.

Proof: A tedious yet straightforward calculation with Theorem 1 completes the proof. ∎

Remark 1 (Why the Perfect Feedback Regimes?): When $\alpha < 2/3$ and $\tilde{\alpha} > 2$, $2\tilde{m}$ indicates the total number of resource levels at the receivers in the backward channel. Hence, one can interpret $2\tilde{m} - \tilde{C}_{\sf pf}$ as the remaining resource levels (resource holes) that can potentially be utilized to aid forward transmission. It turns out feedback can maximize resource utilization by filling up the resource holes underutilized in the non-interactive case. Note that $C_{\sf pf} - C_{\sf no}$ represents the amount of feedback that needs to be sent for achieving $C_{\sf pf}$. Hence, the condition $C_{\sf pf} - C_{\sf no} \le 2\tilde{m} - \tilde{C}_{\sf pf}$ (similarly $\tilde{C}_{\sf pf} - \tilde{C}_{\sf no} \le 2n - C_{\sf pf}$) in Corollary 1 implies that as long as we have enough resource holes, we can get all the way to perfect feedback capacity. We will later provide an intuition as to why feedback can do so while describing our achievability; see Remark 2 in particular. □

IV. Achievability Proof of Theorem 1

We first illustrate our new transmission scheme via a toy example in which the key ingredients of our achievability idea are well presented. Once the description of the scheme is done, we will then outline the proof for generalization while leaving a detailed proof in [12].

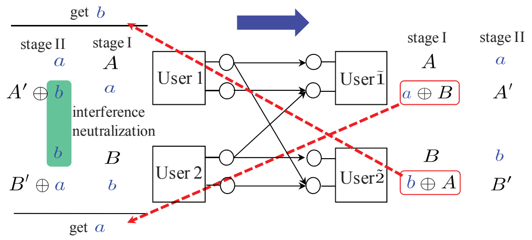

Fig. 4. A perfect feedback scheme for $(n, m) = (2, 1)$ where $C_{\sf pf} = 3$.

A. Example: $(n, m) = (2, 1), (\tilde{n}, \tilde{m}) = (0, 1)$

See Fig. 5 for the channel structure of the example. In this example, Baseline 1 gives $(R, \tilde{R}) = (C_{\sf no}, \tilde{C}_{\sf no}) = (2, 0)$, while Theorem 1 yields $(R, \tilde{R}) = (C_{\sf pf}, \tilde{C}_{\sf pf}) = (3, 1)$.

Perfect feedback scheme: A perfect feedback scheme was presented in [3]. Here we consider a different scheme which allows us to resolve the tension between feedback and independent messages when translated into a two-way scenario. The scheme operates in two stages. See Fig. 4. In stage I, four fresh symbols are transmitted. The scheme in [3] feeds $a \oplus B$ back to user 1, so that user 1 can decode B which turns out to help refining the corrupted symbol a in stage II. On the other hand, here we send $a \oplus B$ back to *user 2*. This way, user 2 can get a by removing its own symbol B. Similarly user 1 can get b. Now in stage II, user 2 intends to re-send b on top, as the b is corrupted due to A in stage I. But here a challenge arises. The challenge is that the b causes interference to user $\tilde{1}$ at the bottom level. But here the symbol b obtained via feedback at user 1 can play a role. The idea of interference neutralization [13] comes into play. User 1 sending the b on bottom enables neutralizing the interference. This then allows user 1 to transmit another fresh symbol, say A', without being interfered. Similarly user 2 can carry B' interference-free. This way, we send 6 symbols during two time slots, thus achieving $C_{\sf pf} = 3$. For the backward IC, on the other hand, we employ the same approach as in [3], taking a relaying idea. User $\tilde{1}$ delivers $\tilde{a}_1$ to user 1 via the feedback-assisted path: user $\tilde{1} \rightarrow$ user 2 $\rightarrow$ feedback $\rightarrow$ user $\tilde{2} \rightarrow$ user 1. Similarly user $\tilde{2}$ sends $\tilde{b}_1$ to user 2. This yields $\tilde{C}_{\sf pf} = 1$. □

We are now ready to describe our achievability. Our scheme operates in two stages. But one noticeable distinction is that each stage comprises a sufficiently large number of time slots. Specifically stage I consists of L time slots, while stage II uses $L+1$ time slots. It turns out our scheme ensures transmission of $6L$ forward symbols and $2L$ backward symbols, thus yielding:

$$(R, \tilde{R}) = \left(\frac{6L}{2L+1}, \frac{2L}{2L+1}\right) \longrightarrow (3, 1) = (C_{\sf pf}, \tilde{C}_{\sf pf}).$$

as $L \to \infty$. Here are details.

Stage I: We employ L time slots. In each time slot, we mimick the perfect feedback scheme while ignoring the tension between feedback and independent message transmissions.

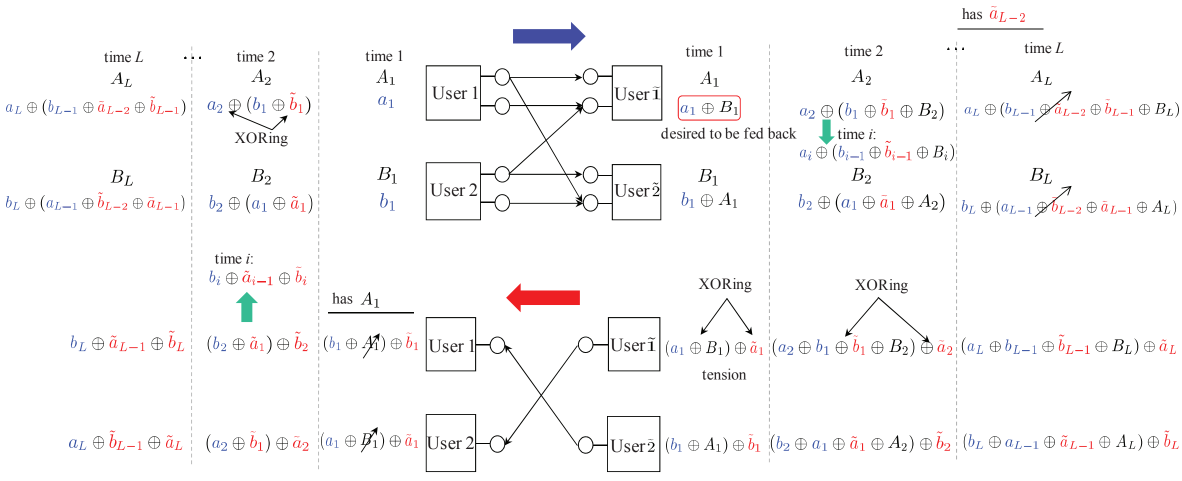

Fig. 5. Stage I: Employ L time slots. The operation in each time slot is similar to stage I's operation in the perfect feedback case. We simply forward the XOR of a feedback signal and a new independent symbol. Here we see the tension between them.

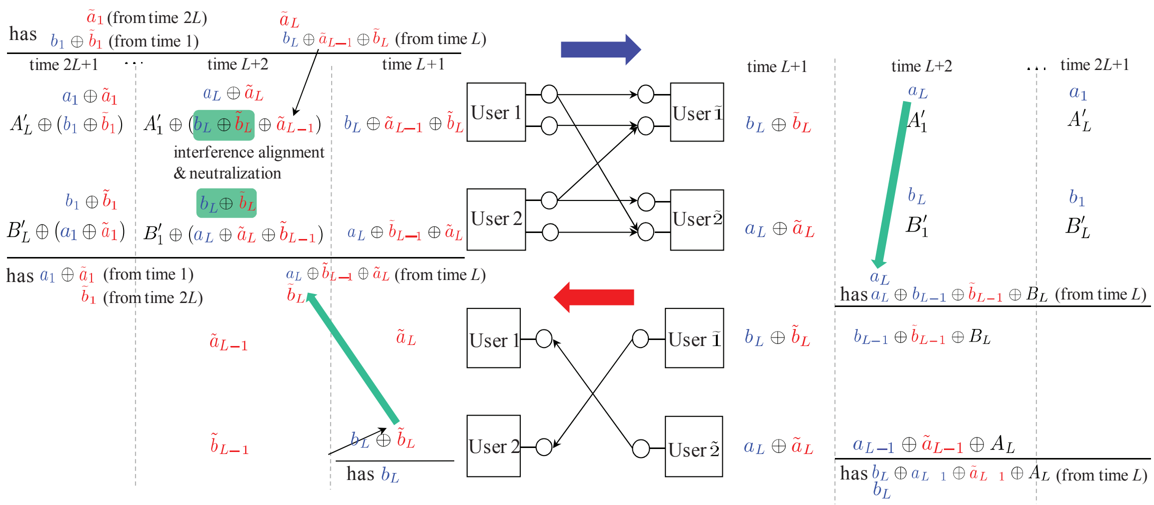

Fig. 6. Stage II: Time $L+1$ aims at decoding $(\tilde{a}_L, \tilde{b}_L)$. At time $L+1+i$, given $(\tilde{a}_{L+1-i}, \tilde{b}_{L+1-i})$ (decoded in time $L+i$), we decode (a_{L+1-i}, b_{L+1-i}) which in turn helping decoding $(\tilde{a}_{L-i}, \tilde{b}_{L-i})$. We iterate this from $i=1$ to $i=L$.

Time 1: Four fresh symbols are transmitted over the forward IC. User $\tilde{1}$ then extracts the one that is desired to be fed back: $a_1 \oplus B_1$. Next we send the XOR of $a_1 \oplus B_1$ and a backward symbol, say $\tilde{a}_1$. Similarly user $\tilde{2}$ transmits $(b_1 \oplus A_1) \oplus \tilde{b}_1$. User 1 then gets $b_1 \oplus \tilde{b}_1$ using its own symbol A_1. Similarly user 2 gets $a_1 \oplus \tilde{a}_1$.

Time 2: User 1 superimposes $b_1 \oplus \tilde{b}_1$ with another new symbol, say a_2, sending the XOR on bottom. On top is another fresh symbol A_2 transmitted. Similarly user 2 sends $(B_2, b_2 \oplus (a_1 \oplus \tilde{a}_1))$. User $\tilde{1}$ transmits $(a_2 \oplus b_1 \oplus \tilde{b}_1 \oplus B_2) \oplus \tilde{a}_2$. Similarly user $\tilde{2}$ sends $(b_2 \oplus a_1 \oplus \tilde{a}_1 \oplus A_2) \oplus \tilde{b}_2$. User 1 then gets $b_2 \oplus \tilde{a}_1 \oplus \tilde{b}_2$ by using its own signal $a_1 \oplus A_2$. Similarly user 2 obtains $a_2 \oplus \tilde{b}_1 \oplus \tilde{a}_2$. Repeating the above, one can readily verify that at time $i \in \{2, \cdots, L\}$, user 1 and 2 get $b_i \oplus \tilde{a}_{i-1} \oplus \tilde{b}_{i-1}$ and $a_i \oplus \tilde{b}_{i-1} \oplus \tilde{a}_{i-1}$ respectively; similarly user $\tilde{1}$ and $\tilde{2}$ get $a_i \oplus b_{i-1} \oplus \tilde{b}_{i-1} \oplus B_i$ and $b_i \oplus a_{i-1} \oplus \tilde{a}_{i-1} \oplus A_i$ on bottom, respectively. See Fig. 5.

Stage II: We employ $L+1$ time slots. We perform refinement w.r.t. the fresh symbols sent in stage I. The novel feature here is that the successive refinement occurs in a *retrospective* manner: the fresh symbol sent at time i is refined at time $2L+2-i$ in stage II where $1 \le i \le L$. Here one key point to emphasize is that the refined symbol in stage II acts as *side information*, which in turn helps refining other past symbols in later time. In the example, the decoding order reads:

$$(\tilde{a}_L, \tilde{b}_L) \to (a_L, b_L) \to \cdots \to (\tilde{a}_1, \tilde{b}_1) \to (a_1, b_1). \quad (5)$$

Time $L+1$: User 1 sends $b_L \oplus \tilde{a}_{L-1} \oplus \tilde{b}_L$ (received at time L) on bottom. It turns out this acts as *ignition* for refining all the corrupted symbols in the past. Similarly user 2 sends $a_L \oplus \tilde{b}_{L-1} \oplus \tilde{a}_L$ on bottom. User $\tilde{1}$ can then obtain $b_L \oplus \tilde{b}_L$ which would be forwarded to user 2. User 2 can then decode $\tilde{b}_L$ of interest. Similarly $\tilde{a}_L$ is delivered to user 1.

Time $L+2$: The decoded symbols $(\tilde{a}_L, \tilde{b}_L)$ turn out to play a key role to refine past forward transmission. Remember that b_L sent by user 2 at time L in stage I was corrupted. User 2 re-transmits the b_L on top as in the perfect feedback case.

But here the problem is that the situation is different from that in the perfect feedback case where b_L was available at user 1 and helped nulling interference. Note that b_L is not available here. Instead user 1 has an interfered version: $b_L \oplus \tilde{b}_L \oplus \tilde{a}_{L-1}$. Nonetheless we can effectively do the same as in the perfect feedback case. User 1 sends $b_L \oplus \tilde{b}_L \oplus \tilde{a}_{L-1}$ on bottom. Clearly the neutralization is not perfect as it contains $\tilde{b}_L$. Here the idea is to exploit the $\tilde{b}_L$ as side information to enable interference alignment and neutralization [14], [15], [13]. Note that user 2 can exploit the knowledge of $\tilde{b}_L$ to construct the *aligned interference* $b_L \oplus \tilde{b}_L$. Sending the $b_L \oplus \tilde{b}_L$ on top, user 2 can completely neutralize the interference as in the perfect feedback case. This enables user 1 to deliver A'_1 interference-free on bottom. Similarly we can deliver (a_L, B'_1) successfully. On the other hand, exploiting a_L (decoded right before) as side information, user $\tilde{1}$ can extract $b_{L-1} \oplus \tilde{b}_{L-1} \oplus B_L$ from the one received at time L. Sending this then allows user 2 to decode $\tilde{b}_{L-1}$. Similarly $\tilde{a}_{L-1}$ can be decoded at user 1.

Time $L+3 \sim$ *Time* $2L+1$: We repeat the same as before. At time $L+1+i$ where $2 \leq i \leq L$, exploiting $(\tilde{a}_{L+1-i}, \tilde{b}_{L+1-i})$ decoded in time $L+i$, we decode (a_{L+1-i}, b_{L+1-i}), which in turn helps decoding $(\tilde{a}_{L-i}, \tilde{b}_{L-i})$.

Now let us compute an achievable rate. In stage I, we sent $(4L, 2L)$ fresh forward and backward symbols. In stage II, we sent only $2L$ fresh forward symbols. This yields the desired rate in the limit of $L \to \infty$.

Remark 2 (Exploiting Future Symbols as Side Information): Note in Fig. 5 the two types of tension: (1) forward-symbol feedback vs. backward symbols; (2) the other counterpart. As illustrated in Fig. 6, our scheme leads us to resolve both tensions. This then enables us to fully utilize the remaining resource level $2\tilde{m} - \tilde{C}_{\mathsf{pf}} = 1$ for sending the forward-symbol feedback of $C_{\mathsf{pf}} - C_{\mathsf{no}} = 1$, thereby achieving C_{pf}. Similarly we can fill up the resource holes $2n - C_{\mathsf{pf}} = 1$ with the backward-symbol feedback of $\tilde{C}_{\mathsf{pf}} - \tilde{C}_{\mathsf{no}} = 1$. This comes from the fact that our feedback scheme exploits the following as *side information*: (i) past signals; (ii) users' own traffic; (iii) partially decoded symbols. While the first two were already shown to be beneficial in the prior works [3], [5], the third type of information is the newly exploited one which turns out to yield the strong interaction gain. One can view this as *future* information. Recall the decoding order (5). When decoding $(\tilde{a}_{L-1}, \tilde{b}_{L-1})$, we exploited (a_L, b_L) (future symbols w.r.t. $(\tilde{a}_{L-1}, \tilde{b}_{L-1})$) as side information. A conventional belief is that feedback allows us to know only about the *past*. In contrast, we discover a new viewpoint on the role of feedback. Feedback enables exploiting future information as well via retrospective decoding. □

B. Proof Outline

We categorize regimes depending on the values of channel parameters. Notice that $\mathcal{C} = \mathcal{C}_{\mathsf{no}}$ when $(\alpha \in [\frac{2}{3}, 2], \tilde{\alpha} \in [\frac{2}{3}, 2])$. Also by symmetry, it suffices to consider only five regimes: (R1) $\alpha > 2, \tilde{\alpha} > 2$; (R2) $\alpha \in (0, 2/3), \tilde{\alpha} \in (0, 2/3)$; (R3) $\alpha > 2, \tilde{\alpha} \in [2/3, 2]$; (R4) $\alpha \in (0, 2/3), \tilde{\alpha} \in [2/3, 2]$; (R5) $\alpha \in (0, 2/3), \tilde{\alpha} > 2$. As figured out in Fig. 2, (R1) and (R2) are the ones in which there is no interaction gain. The proof builds only upon the perfect feedback scheme [3]. (R3) and (R4) are the ones in which there is an interaction gain but only in one direction. A non-trivial approach is employed for feedback (e.g., w.r.t. forward transmission) to gracefully coexist with the backward traffic. It turns out part of the ideas illustrated in the example leads to the claimed rate region. (R5) is the one in which there is an interaction gain and sometimes one can get to perfect feedback capacities. We fully utilize the ideas presented in the example to prove the claimed rate region. See [12] for a detailed proof.

V. Discussion

The channel asymmetry, which yields an interaction gain (see Fig. 2), often occurs in broadband systems where there are many subchannels exhibiting rich diversity on channel gains. Hence, pairing appropriate forward and backward ICs, one can easily obtain such a significant gain. Our future work includes: (1) Translating to the Gaussian channel; (2) Discovering other two-way scenarios in which one can achieve a huge interaction gain; (3) Generalizing our new achievability to broader network contexts.

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