A Relay Can Increase Degrees of Freedom in Bursty Interference Networks

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***Abstract*— We investigate the benefits of incorporating relays in future multi-user wireless networks that seek to exploit unexplored bands of very high frequency spectrum, where transmitted signals are known to be highly susceptible to outages. To this end, we examine a two-user *bursty* MIMO Gaussian interference channel with an in-band relay, where Bernoulli random states conceptually capture signal outages. As our main result, we show that an in-band relay can provide a degrees of freedom (DoF) gain in this bursty channel. This beneficial role of in-band relays in the bursty channel is in direct contrast to their role in the non-bursty channel which is not as significant to provide a DoF gain. More importantly, we demonstrate that in certain antenna configurations, an in-band relay can help achieve *interference-free* performances with increased DoF. We find the benefits particularly substantial in high-outage circumstances, as the DoF gain can grow *linearly* with the number of antennas at the relay. In this paper, first we derive an outer bound from which we obtain a necessary condition for interference-free DoF performances. Then we develop a novel scheme that exploits information of the bursty channel states to achieve them.**

***Index Terms*— Bursty interference, relay networks, interference cancellation, degrees of freedom.**

I. Introduction

MOBILE data demands are on the rise at an alarming rate. This has led us to a point where improving spectral efficiency of the spectrum bands used in the current wireless networks has become of little gain. To cope with it, unexplored spectrum bands in very high frequency (so-called millimeter wave bands, mmWave bands for short) are attracting attention. Naturally, the performance limit of the mmWave channel is of great interest to many.

In characterizing the limit, the conventional information-theoretic view may not be appropriate. This is due to the drastically different physical characteristics of the mmWave channel. Its very high carrier frequency gives rise to a great deal of path-loss and increased directivity, and as a result, mmWave signals are known to be highly susceptible to signal outages [1], [2]. A new perspective is thus required in investigating the limit of mmWave channels, and we set out to pursue it in this paper.

Conceptually, we can capture mmWave signal outages by considering channels where a transmitted signal either is received by a receiving node with probability p, or results in an outage with probability $1-p$. This simple modeling leads us to observe an interesting phenomenon, which sticks out in interference channels where from the traditional point of view, interference is present at all times. From the new perspective, by contrast, it is not. Its presence now bears a *bursty* nature, and this can open up new opportunities to mitigate interference that has long been a major barrier in wireless communication.

In this paper, we examine whether in-band relays can play a key role in mitigating *bursty* interference and provide significant gains in multi-user wireless networks. Although past work [3] found the role of in-band relays to be pessimistic in interference channels as they provide no degrees of freedom (DoF) gain, our work on bursty interference channels tells a different story. The motivation of our work comes from an observation in a simple single-user channel, where we can anticipate promising benefits that relays can bring into networks showing bursty behaviors.

To see this, consider a *bursty* MIMO Gaussian relay channel, where the signal of the transmitter is received by either both the receiver and the relay with probability p, or none with probability $1-p$, and the signal of the relay is always received by the receiver; the transmitter has a large number of antennas, and the receiver and the relay have 1 and L respectively. By the standard cut-set argument, we obtain an outer bound on the DoF: $\min\{p(1+L), 1\}$, which is also achievable as the cut-set bound is well-known to be tight in single-source single-destination networks [4]. Observe that the DoF is p without a relay ($L=0$), and it becomes strictly larger with a relay ($L \geq 1$). From this observation, we see that an in-band relay in the single-user bursty channel can provide a DoF gain. Moreover, we see that the gain can grow *linearly* with L in high-outage regimes ($p \ll 1$).

To provide an intuitive explanation, the DoF gain comes from the receive-forward operation of the relay; it receives L extra symbols from the transmitter when the channel is stable, and forwards them to the receiver in a first-in-first-out (FIFO) manner when the channel is unstable. The relay exploits unstable moments of the channel and provides the receiver with useful symbols.

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The benefits that a relay brings into the single-user bursty channel motivate us to further examine whether they carry over into multi-user bursty interference channels. Particularly interested in possibilities that a relay can mitigate bursty interference between multiple users to a considerable extent and offer the benefits to each user as if *no* interference is present, we ask: can relays play a significant role in bursty interference channels to help achieve *interference-free* DoF performances?

To answer this question, we consider a two-user *bursty* MIMO Gaussian interference channel (IC) with an in-band relay, where Bernoulli random states represent signal outage behaviors at receiving nodes. It would be a thorough investigation of the mmWave channel to consider every link between a transmitting node and a receiving node to be subject to signal outages. However, as initial efforts into it, we consider a simpler channel in this paper; the signal of a transmitter is received by either all the receivers and the relay (all receiving nodes) with probability p, or none with probability $1-p$, and the signal of the relay is always received by all the receivers. That is, we incorporate two Bernoulli random states, each of which captures the outage behaviors of each transmitter's signals at all receiving nodes. From a practical point of view, this channel can reflect a scenario where we place the relay close to the receivers to cope with severe signal outages the receivers experience [5]. Since the relay and the receivers are in close proximity, the outage occurrences of transmitted signals of each transmitter can be considered highly coupled, and transmitted signals of the relay can be considered rarely prone to outages at the receivers.

As our main results, in this channel, we derive an outer bound from which we obtain a necessary condition on the antenna configuration for interference-free DoF performances. Further, we develop a novel scheme that harnesses information of the bursty channel states. Through this information, our scheme enables the relay and the transmitters to cooperate in a beneficial fashion, and provides a significant DoF gain over the channel without a relay. More importantly, our scheme reveals that an in-band relay can indeed help achieve *interference-free* performances with increased DoF. We find the presence of a relay particularly beneficial in high-outage circumstances, as the DoF gain can grow *linearly* with the number of antennas at the relay. Our results show that the role of in-band relays in the bursty channel is crucial in constrast to their role in the non-bursty channel where in-band relays provide no DoF gain [3].

Considerable work has been done toward understanding ICs with relays. Numerous techniques developed in the IC and the RC have been combined and applied to ICs with various relay types. Although we focus on in-band relays, out-of-band relays have also been of interest. Sahin *et al.* [6] and Tian and Yener [7] considered an IC with an out-of-band relay that receives and transmits in a band orthogonal to the IC. Sridharan *et al.* [8] considered an IC with an out-of-band reception and in-band transmission relay.

In-band relays have drawn great attention. Sahin and Erkip [9] first considered an IC with an in-band relay, and proposed an achievable scheme that employs a

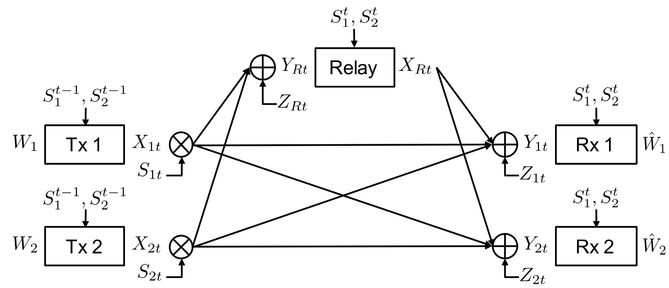

Fig. 1. Bursty MIMO Gaussian interference channel with an in-band relay.

decode-forward scheme [10] and rate-splitting [11]. The relay decodes common and private messages, and forwards them by allocating its power across them. Marić *et al.* [12] suggested a different decode-forward based achievable scheme. The relay intentionally forwards interfering signals to enhance the reception of interference at the receivers, so that interference cancellation can be facilitated. Other works [13]–[17] mostly proposed achievable schemes that extend a compress-forward scheme [10]. In the schemes, the relay forwards compressed descriptions of its received signals, and the receivers decode messages based on their own received signals and the descriptions.

A distinction of our work compared to all past works is that we investigate future wireless networks seeking to utilize mmWave spectrum bands, where signal outages should be emphasized due to the distinct physical characteristics of the spectrum bands in question. Some past works, having different motivations, investigated models that bear a similar bursty nature. Vahid *et al.* [18] studied a binary fading IC where signals and channel gains are in the binary field, and the channel gains are random variables. Wang and Diggavi [19] studied a bursty Gaussian IC without a relay. Extending the results in [19] to the MIMO channel with an in-band relay, we demonstrate that relays can offer substantial benefits in the bursty MIMO Gaussian IC.

II. Problem Formulation

A. Model Description

Fig. 1 describes the bursty MIMO Gaussian interference channel (IC) with an in-band relay. The transmitters, the receivers, and the relay have M, N, and L antennas, respectively. Transmitter k wishes to deliver message W_k reliably to receiver k, $\forall k = 1, 2$. Let $X_{kt} \in \mathbb{C}^M$ be the encoded signal of transmitter k at time t, and $X_{Rt} \in \mathbb{C}^L$ be the encoded signal of the relay at time t. We introduce two multiplicative random states S_{kt}, one intended for each transmitter, to conceptually represent signal outages caused by the severe path-loss and directivity of mmWave signals. We assume S_{kt} to be independent, Bern(p) and i.i.d. over time. The relay is not subject to signal outages, thus not restricted by such a random state. This way of constructing a bursty channel is to first consider a simple mmWave channel as initial efforts in investigating it, where the signal of a transmitter is received by either all the receivers and the relay, or none, which can be the consequence of placing the relay close to the receivers in practice. Additive noise terms Z_{kt} and Z_{Rt} are assumed to be independent, $\mathcal{CN}(0, \mathbf{I}_N)$ and $\mathcal{CN}(0, \mathbf{I}_L)$, and i.i.d. over time. Let $Y_{kt} \in \mathbb{C}^N$ be the received signal of receiver k at time t, and $Y_{Rt} \in \mathbb{C}^L$ be the received signal of the relay at time t:

$$Y_{kt} = \mathbf{H}_{k1} S_{1t} X_{1t} + \mathbf{H}_{k2} S_{2t} X_{2t} + \mathbf{H}_{kR} X_{Rt} + Z_{kt},$$
$$Y_{Rt} = \mathbf{H}_{R1} S_{1t} X_{1t} + \mathbf{H}_{R2} S_{2t} X_{2t} + Z_{Rt}.$$

The matrices $\mathbf{H}_{ji}$, $\mathbf{H}_{Ri}$, and $\mathbf{H}_{jR}$ describe the time-invariant channels from transmitter i to receiver j, from transmitter i to the relay, and from the relay to receiver j, $\forall i, j = 1, 2$. All channel matrices are assumed to be full rank.

Current channel states are available at the receivers and the relay, since the receiving nodes can detect which transmitter's signal is traversing a stable channel, for instance, by measuring the energy levels of incoming signals. In the rest of this paper, we say that a transmitter is active when its transmitted signal travels through a stable mmWave channel, thus is received by all the receiving nodes without an outage. Also, we assume the transmitters get feedback of past channel states from the receivers. Through this feedback, the uncoordinated transmitters can devise ways to cooperate. Thus, transmitter k generates its encoded signal at time t based on its own message and the feedback of past channel states:

$$X_{kt} = f_{kt}(W_k, S^{t-1}).$$

Shorthand notation S_t stands for (S_{1t}, S_{2t}) and S^{t-1} stands for the sequence up to $t-1$. The relay generates its encoded signal at time t based on its past received signals, and both past and current channel states:

$$X_{Rt} = f_{Rt}(Y_R^{t-1}, S^t).$$

Shorthand notation Y_R^{t-1} stands for the sequence up to $t-1$.

We define the DoF region as follows:

$$\mathcal{D} = \left\{ (d_1, d_2) : \begin{array}{l} \exists (R_1, R_2) \in \mathcal{C}(P) \text{ such that} \\ d_k = \lim_{P\to\infty} \dfrac{R_k}{\log P} \end{array} \right\}.$$

$\mathcal{C}(P)$ is the capacity region with power constraint P on each antenna, defined as the closure of the set of rate pairs (R_1, R_2) that are achievable by a sequence of codes $(2^{nR_1}, 2^{nR_2}, n)$. We follow the conventional way of defining the DoF regions of non-bursty channels. Another way to define the DoF regions of bursty channels would be to divide rate tuples in the capacity region by $p \log(P)$, to observe how the rate tuples scale in comparison to the capacity of a bursty Gaussian point-to-point channel at high signal-to-noise ratio ($P \to \infty$). However, when our main interest is to compare the DoF regions of the Gaussian ICs with and without a relay, the new definition makes little difference. To fairly observe the effect of a relay, we keep the parameter of bursty channel state p constant, and the new definition only expands or shrinks the two DoF regions by the same factor. In this regard, we follow the convention in this work.

It is also important to examine power-constrained regimes ($P < \infty$), as they concern more practically relevant networks. We note, however, that exploring whether adding a relay can be useful in terms of DoF can provide an opportunity to gain insights into the benefits of employing relays into real-world networks at a high level of abstraction. Considering power-unconstrained regimes ($P \to \infty$), we can investigate how many conceptually independent data streams (one stream being associated with the communication in the point-to-point channel) we can reliably convey in the channel of interest.

III. Main Results

For completeness, we first describe the following result for the single-user case, which is immediate since the cut-set bound is tight in terms of DoF in single-user networks [4].

Theorem 1: The DoF of the bursty MIMO Gaussian relay channel is characterized by

$$d = \min \left\{ \begin{array}{l} p \min(M, N+L), \\ p \min(M+L, N) + (1-p)\min(L, N) \end{array} \right\}.$$

Next, we present our main results for the bursty MIMO Gaussian interference channel (IC) with an in-band relay.

Theorem 2: A DoF outer bound of the bursty MIMO Gaussian IC with an in-band relay is

$$d_1, d_2 \le \min \left\{ \begin{array}{l} p \min(M, N+L), \\ p \min(M+L, N) \\ +(1-p)\min(L, N) \end{array} \right\}, \quad (1)$$

$$d_1 + d_2 \le \min \left\{ \begin{array}{l} p \min\left\{(M-N)^+, N+L\right\}, \\ p \min\left\{(M+L-N)^+, N\right\} \\ +(1-p)\min\left\{(L-N)^+, N\right\} \end{array} \right\} + \left\{ \begin{array}{l} p^2 \min(2M+L, N) \\ +2p(1-p)\min(M+L, N) \\ +(1-p)^2 \min(L, N) \end{array} \right\}. \quad (2)$$

Proof: See Section V. ■

We note that the above bound recovers the DoF results for the non-bursty case ($p = 1$) [3] and the case without a relay ($L = 0$) [19].

Using this bound, we obtain a necessary condition for attaining interference-free DoF. This is done by examining when (2) becomes inactive. The proof is in Appendix A.

Corollary 1: A necessary condition for attaining interference-free DoF is the union of three conditions $\mathcal{C}_1$, $\mathcal{C}_2$, and $\mathcal{C}_3$ below:

$$\mathcal{C}_1 : 2M \le N,$$
$$\mathcal{C}_2 : M \ge 2N + L \text{ and } L \ge 2N,$$
$$\mathcal{C}_3 : M \ge 2N \text{ and } 3L \le N.$$

Finally, we establish a sufficient condition for attaining interference-free DoF.

Theorem 3: A sufficient condition for attaining interference-free DoF is the union of three conditions $\mathcal{C}_1, \mathcal{C}_2$ above, and $\mathcal{C}_3'$ below:

$$\mathcal{C}_3' : M \ge 2N + L \text{ and } 3L \le N.$$

Proof: See Section IV. ■

We briefly outline the schemes to be presented.

- $\mathcal{C}_1$: Each receiver can decode all symbols sent by both transmitters. The relay sends nothing at all times.

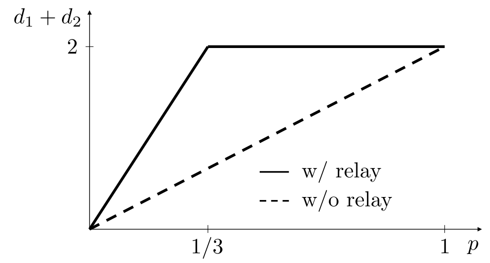

Fig. 2. Sum DoF of the bursty MIMO Gaussian IC with and without an in-band relay. $(M, N, L) = (4, 1, 2)$ and $(M, N, L) = (4, 1, 0)$.

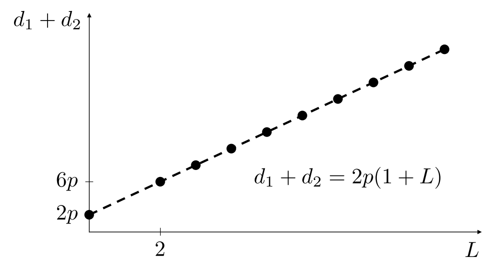

Fig. 3. Linear scalability of the sum DoF of the bursty MIMO Gaussian IC with an in-band relay in high-outage regimes $p \ll 1$. $M = \infty$, $N = 1$, and $L \geq 2$.

- $\mathcal{C}_2$: All transmitting nodes (the transmitters and the relay) can apply zero-forcing precoding in sending signals toward their corresponding receiving nodes. They can send symbols separately to each receiving node so that each receiving node does not get undesired symbols. But since the relay is shared by both users, the relay gets collided symbols when both transmitters are active at the same time. In our scheme, the relay cooperates with an active transmitter when the relay forwards these collided symbols to the receivers. This cooperation removes the interference in the collided symbols and delivers only the desired symbols to the receivers.
- $\mathcal{C}_3'$: The transmitters can apply zero-forcing precoding, but the relay cannot. Since the relay cannot send symbols separately to each receiver, unavoidable collisions take place at the receivers. In our scheme, each transmitter provides the other receiver with side information so that the receivers exploit it to resolve the interference.

Fig. 2 illustrates the sum DoF of the bursty MIMO Gaussian IC with and without an in-band relay. We compare two antenna configurations $(M, N, L) = (4, 1, 2)$ and $(M, N, L) = (4, 1, 0)$. We can observe that the relay offers a DoF gain. Fig. 3 illustrates linear scalability of the sum DoF of the bursty MIMO Gaussian IC with an in-band relay in high-outage regimes $p \ll 1$. We consider a class of antenna configurations in which the relay has multiple antennas: $M = \infty$, $N = 1$, and $L \geq 2$. We can observe that the sum DoF grows linearly with the number of antennas at the relay.

In this work, our results do not characterize the DoF region of the bursty MIMO Gaussian IC with an in-band relay. Our main focus is to discuss interference-free DoF performances and to show their optimality. That being said, we can establish the DoF region of the bursty SISO Gaussian IC with an in-band multi-antenna relay by our inner and outer bound results, thus we state it in the following theorem:

Theorem 4: The DoF region of the bursty SISO Gaussian IC with a multi-antenna relay is

$$\mathcal{D} = \Big\{(d_1, d_2) : d_1, d_2 \leq p,\ d_1 + d_2 \leq \min(2p, 1)\Big\}.$$

Proof: See Appendix D. ∎

IV. Proof of Theorem 3

In this section, we develop an explicit scheme that achieves interference-free DoF in the bursty MIMO Gaussian IC with an in-band relay. We distinguish two different regimes depending on how severe signal outages are.

- High-outage regime: Frequent signal outages limit the information flow at the transmitters. Thus, the transmitters send as much information as possible per active transmission toward the intended receivers and the relay.
- Low-outage regime: Infrequent signal outages may limit the information flow at the receivers, especially when they have a small number of antennas. In this case, the transmitters reduce the amount of information sent per active transmission to ensure decoding of the information at the intended receivers.

A. $2M \leq N$

Each transmitter sends M fresh symbols at all times, and the relay sends nothing. Since each receiver has a sufficient number of antennas, it decodes its desired symbols whenever its corresponding transmitter is active. This scheme achieves the following DoF region.

$$\mathcal{D} = \Big\{(d_1, d_2) : d_1, d_2 \leq pM\Big\}.$$

B. $M \geq 2N + L$ and $L \geq 2N$

We present the scheme with an example for the simplest antenna configuration: $(M, N, L) = (4,1,2)$. Fig. 4 demonstrates how the transmitters and the relay operate with a sequence of channel states (S_1, S_2): (1,1), (0,1), (1,0), (0,0). The generalization of the scheme is in Appendix B with a special case to note, which is a series of successive (1,1) channel states. The transmitters and the relay always apply zero-forcing precoding. The connected links in Fig. 4 depict the effect of zero-forcing precoding.

1) High-Outage Regime $p < \frac{1}{3}$: Each transmitter sends one fresh symbol to its intended receiver and two to the relay. The use of one extra antenna is to send symbols to the other receiver when cooperation with the relay is needed.

- Time 1: Both transmitters send three fresh symbols. Knowing both are active through information of the current channel states, the relay sends nothing to avoid interference at the receivers. However, two unavoidable collisions take place at the relay: $a_1 + b_1$ and $a_2 + b_2$.
- Time 2: From feedback, the transmitters are aware of the past collisions at the relay. In addition to three fresh symbols toward receiver 2 and the relay, transmitter 2

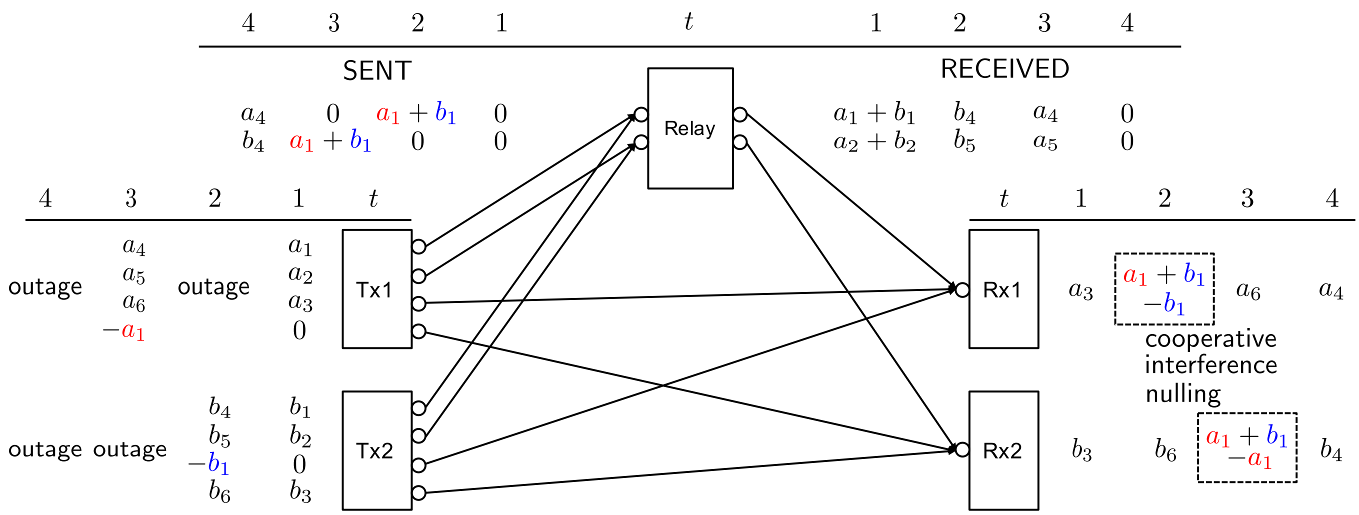

Fig. 4. An achievable scheme for $(M, N, L) = (4, 1, 2)$ configuration.

sends $-b_1$ to receiver 1 in hopes of removing its footprint in one of the past collisions, $a_1 + b_1$. Knowing that transmitter 2 is trying to deal with the collision, the relay sends $a_1 + b_1$ to receiver 1 and nothing to receiver 2. Receiver 1 gets a_1, the sum of $a_1 + b_1$ from the relay and $-b_1$ from transmitter 2. The relay and transmitter 2 *cooperate* and deliver only desired symbol a_1 to receiver 1 with interference b_1 removed.

- Time 3: In addition to three fresh symbols, transmitter 1 sends $-a_1$ to receiver 2. The relay sends $a_1 + b_1$ to receiver 2 and nothing to receiver 1. The cooperation between the relay and transmitter 1 delivers b_1 to receiver 2.
- Time 4: Both transmitters are inactive, and the relay knows there is no active transmitter to cooperate with. The relay delivers two past reserved symbols that were not collided. It sends a_4 to receiver 1 and b_4 to receiver 2.

The proposed scheme works when relay-passing symbols, such as a_1, b_1, a_4, and b_4, are delivered to the intended receivers faster than they build up at the relay. Let us perform a simple analysis that compares the rate at which relay-passing symbols build up and the rate at which they are delivered. By the symmetry of the scheme, it suffices to carry out the analysis from user 1's perspective. The analysis deals with two types of user 1's relay-passing symbols.

- Type 1 symbols: collisions (a_1, a_2). Two symbols, collided with symbols of the other user, are reserved at the relay with probability p^2 (Time 1: both transmitters are active.), and one of them can be delivered to receiver 1 through *cooperation* between the relay and transmitter 2 with probability $(1-p)p$ (Time 2: only transmitter 2 is active.). Type 1 symbols are delivered faster than they build up at the relay when the following holds.

$$p^2 \times 2 < (1-p)p \times 1.$$

- Type 2 symbols: collision-free (a_4, a_5). Two symbols, without being collided, are reserved at the relay with probability $p(1-p)$ (Time 3: only transmitter 1 is active.), and one of them can be delivered to receiver 1 by the relay with probability $(1-p)^2$ (Time 4: both transmitters are inactive.). Type 2 symbols are delivered faster than they build up at the relay when the following holds.

$$p(1-p) \times 2 < (1-p)^2 \times 1.$$

In the high-outage regime where $p < \frac{1}{3}$, the above conditions hold. Each transmitter sends 3 fresh symbols at the rate of p due to outages, and all of them will be eventually decoded at the intended receiver. This achieves the individual DoF of $3p$.

2) Low-Outage Regime $\frac{1}{3} \leq p < 1$: Bothtransmitters send symbols at a *lower* rate; each transmitter chooses to send symbols with probability q at any time instant. Each transmitter makes such decisions independently over time, and the decisions of the transmitters are independent. Therefore, each transmitter is in fact active with probability pq. A similar analysis by replacing p with pq gives the following conditions.

- Type 1 symbols: $(pq)^2 \times 2 < (1-pq)(pq) \times 1$.
- Type 2 symbols: $(pq)(1-pq) \times 2 < (1-pq)^2 \times 1$.

In the low-outage regime where $\frac{1}{3} \leq p < 1$, defining q as $\frac{1}{p}(\frac{1}{3} - \epsilon)$, where $\epsilon > 0$, satisfies the above conditions. Each transmitter sends 3 fresh symbols at the rate of pq, and all of them will be eventually decoded at the intended receiver. This achieves the individual DoF of $3pq$. As both transmitters choose ϵ arbitrarily close to zero, the individual DoF converges to 1. In summary, the proposed scheme achieves the following DoF region.

$$\mathcal{D} = \left\{(d_1, d_2) : d_1, d_2 \leq \min(3p, 1)\right\}.$$

Remark 1 (Cooperative Interference Nulling): From user 1's perspective, to achieve interference-free DoF performances, transmitter 1 should always send three fresh symbols as in the single-user case. One of them is directly delivered to receiver 1, and the other two are reserved at the relay and delivered later when transmitter 1 is inactive.

Unfortunately, since the relay is shared, the relay-passing symbols of user 1 sometimes get interfered with those of user 2. But, cooperative interference nulling removes the interference in the user 1's relay-passing symbols and delivers only the desired symbols to receiver 1. At Time 2, for example, when transmitter 1 is inactive, the relay and active transmitter 2 cooperate and remove interference b_1 in $a_1 + b_1$ to deliver desired symbol a_1 to receiver 1. When both transmitters are inactive, the relay applies zero-forcing precoding and delivers user 1's relay-passing symbols that were not interfered, for example, a_4 at Time 4. Overall, the operation coincides with the single-user scheme: transmitter 1 always sends three symbols. One of them is directly delivered to receiver 1, and the other two are delivered through the relay to receiver 1 without interference.

Remark 2 (Distinction From Other Relaying Schemes): Cooperative interference nulling is a notable distinction from other relaying schemes. It provides a DoF gain in the bursty IC, although other relaying schemes provide only power gains in the non-bursty IC. In cooperative interference nulling, the relay and the transmitters *synchrounously* cooperate by exploiting information of the bursty channel states, to remove interference in their signals in the air and deliver only desired signals to the receivers. In other schemes based on decode-forward strategies [9], [12], the relay and the transmitters also cooperate by generating their signals coherently. However, the schemes provide only power gains. In other schemes mostly based on compress-forward strategies [13]–[17], the relay forwards additional descriptions of its received signals to help decoding at the receivers without cooperating with the transmitters. Also, the schemes provide only power gains.

Remark 3: (Why There Is a DoF Gain in the Bursty Case, But None in the Non-Bursty Case): In the bursty case, transmitters are sometimes stuck in severely fading channels thus prone to signal outages. In these moments, the presence of a relay is valuable as it can be a reliable temporary information source for the receivers. Based on its past received symbols, the relay can either send symbols that can help resolve past collisions at the receivers, or send fresh symbols that were reserved at the relay and have not been delivered to the receivers. In both ways, the relay enables a receiver or both to decode extra symbols while a transmitter or both cannot due to outages. These extra symbols amount to a DoF gain. In the non-bursty case, however, transmitters are not susceptible to signal outages. It means that whatever symbols the relay can possibly deliver to the receivers based on its past received symbols, the transmitters can deliver the same symbols to the receivers by themselves: be it symbols that can help resolve past collisions, or fresh symbols. The relay finds no moments to be as valuable as it can be in the bursty case where transmitters sometimes cannot send useful symbols at all stuck in severely fading channels.

C. $M \geq 2N + L$ and $3L \leq N$

We present the scheme with an example for the simplest antenna configuration: $(M, N, L) = (7,3,1)$. Fig. 5 demonstrates how the transmitters and the relay operate with a sequence of channel states (S_1, S_2): (1,1), (0,1), (1,0), (0,0). The generalization of the scheme is in Appendix C. The transmitters always apply zero-forcing precoding, whereas the relay cannot. The connected links in Fig. 5 depict the effect of zero-forcing precoding.

1) High-Outage Regime $p < \frac{1}{2}$: Each transmitter sends three fresh symbols to its intended receiver and one to the relay. The use of three extra antennas is to provide the other receiver with side information that is needed to resolve unavoidable collisions.

- Time 1: Both transmitters send four fresh symbols. In addition, each transmitter sends to the other receiver the duplicate of its relay-passing symbol. This is to provide side information. There is one collision at the relay, $a_1 + b_1$, and one at each receiver, $a_2 + b_1$ and b_2+a_1. For each receiver, this collision has to be resolved to decode its desired symbol, a_2 and b_2.
- Time 2: In addition to four fresh symbols, transmitter 2 sends b_1 and b_5, the duplicate of its relay-passing symbols, to receiver 1. This is again to provide side information. The relay broadcasts $a_1 + b_1$ to deliver a_1 to receiver 1 whose corresponding transmitter is inactive. Receiver 1 decodes a_1, b_1, and b_5. Receiver 1 resolves past collision $a_2 + b_1$ with *side information* b_1. b_5 will be used later.
- Time 3: In addition to four fresh symbols, transmitter 1 sends a_1 and a_5 to receiver 2. The relay broadcasts $a_1 + b_1$. Receiver 2 decodes b_1, a_1, and a_5. Receiver 2 resolves past collision $b_2 + a_1$ with side information a_1. a_5 will be used later.
- Time 4: Both transmitters are inactive. The relay sends the sum of a_5 and b_5 to deliver information that is useful for both receivers. From $a_5 + b_5$, receiver 1 decodes a_5 since it has b_5 as side information, and receiver 2 decodes b_5 since it has a_5 as side information.

The proposed scheme works when relay-passing symbols, such as a_1, b_1, a_5, and b_5, are delivered to the intended receivers faster than they build up at the relay. Also, each receiver needs all relay-passing symbols of the other user as side information, because they are broadcast by the relay and cause interference. This is why each transmitter keeps trying to provide the other receiver with side information at the cost of unnecessary interference, for example, at Time 1. Let us perform a simple analysis. First, we compare the rate at which relay-passing symbols build up and the rate at which they are delivered. Second, we examine the rate at which side information is provided. Due to the symmetry of the scheme, it suffices to carry out the analysis from user 1's perspective.

- User 1's relay-passing symbols (a_1, a_5): one symbol, possibly collided with a symbol of the other user, reserved at the relay with probability p (Time 1, 3: transmitter 1 is active.), and it can be delivered to receiver 1 by the relay with probability $1 - p$ (Time 2, 4: transmitter 1 is inactive.). Transmitted symbols of the relay can be in the form of sums of both users' relay-passing symbols. Yet, receiver 1 can decode user 1's relay-passing symbols since transmitter 2 provides side information properly.

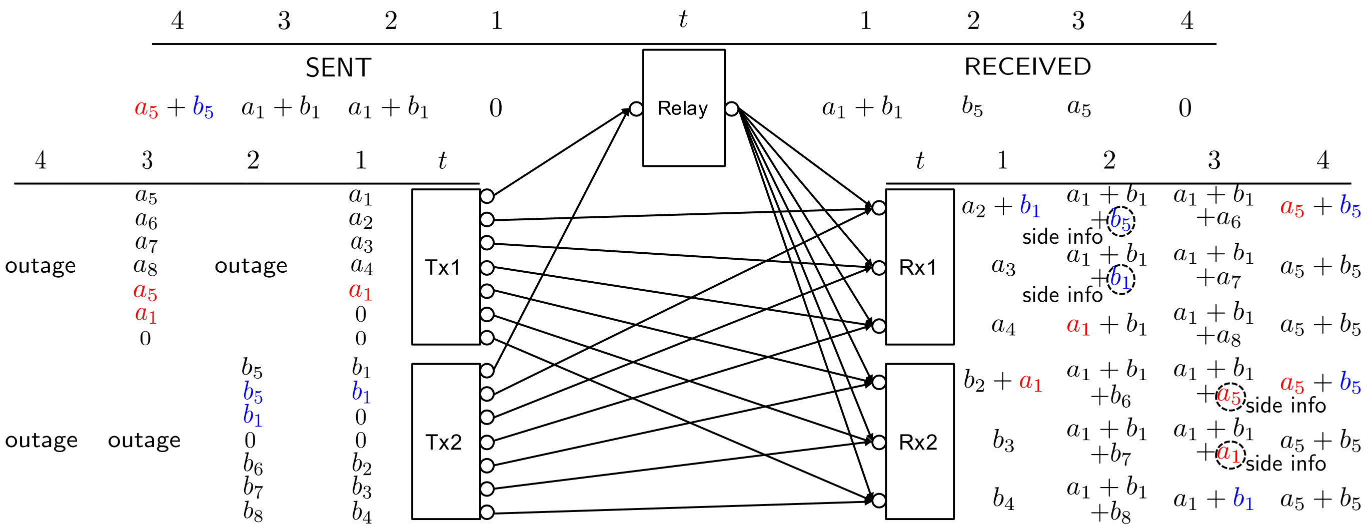

Fig. 5. An achievable scheme for $(M, N, L) = (7,3,1)$ configuration.

This matter is discussed in the next item. Considering user 1's relay-passing symbols, they are delivered faster than they build up at the relay when the following holds.

$$p \times 1 < (1 - p) \times 1.$$

- User 2's relay-passing symbols (b_1, b_5): one symbol is reserved at the relay with probability p (Time 1, 2: transmitter 2 is active.), and eventually broadcast. This causes interference at receiver 1. Receiver 1 can get the duplicate of at most two user 2's relay-passing symbols as *side information* from transmitter 2 with probability $(1 - p)p$ (Time 2: only transmitter 2 is active.). The reason why receiver 1 gets at most two of them is that it uses one antenna to get one user 1's relay-passing symbol from the relay, and has two antennas left. Receiver 1 exploits this side information to resolve the interference caused by the broadcasting of the relay. Side information from transmitter 2 is provided at a faster rate than the rate at which user 2's relay-passing symbols build up at the relay when the following holds.

$$p \times 1 < (1 - p)p \times 2.$$

In the high-outage regime where $p < \frac{1}{2}$, the above conditions hold. Each transmitter sends 4 fresh symbols at the rate of p due to outages, and all of them will be eventually decoded at the intended receiver in the high-outage regime. This achieves the individual DoF of $4p$.

2) Low-Outage Regime $\frac{1}{2} \le p < 1$: Both transmitters send fresh symbols to the relay at a *lower* rate; they choose to send symbols to the relay with probability q at any time instant. Each transmitter makes such decisions independently over time, and the decisions of the transmitters are independent. Therefore, each transmitter sends symbols to its intended receiver at the rate of p, and to the relay at the rate of pq. We can perform a similar analysis from user 1's perspective.

- User 1's relay-passing symbols are reserved at the rate of $pq \times 1$, and they can be delivered at the rate of $(1-p)\times 1$.

$$pq \times 1 < (1 - p) \times 1.$$

- User 2's relay-passing symbols are reserved at the rate of $pq \times 1$, and eventually broadcast. Receiver 1 can get side information at the rate of $(1 - p)p \times 2$.

$$pq \times 1 < (1 - p)p \times 2.$$

In the low-outage regime where $\frac{1}{2} \le p < 1$, defining q as $\frac{1}{p}(1 - p - \epsilon)$, where $\epsilon > 0$, satisfies the above conditions. Each transmitter sends 3 fresh symbols to its intended receiver at the rate of p, and 1 fresh symbol to the relay at the rate of pq. All of them will be eventually decoded at the intended receiver. This achieves the individual DoF of $3p + 1pq$. As both transmitters choose ϵ arbitrarily close to zero, the individual DoF converges to $2p + 1$. In summary, the proposed scheme achieves the following DoF region.

$$\mathcal{D} = \Big\{(d_1, d_2) : d_1, d_2 \le \min(4p, 2p + 1)\Big\}.$$

Remark 4 (Exploiting Side Information): The limited number of antennas at the relay disables cooperative interference nulling. Moreover, transmitted symbols of the relay are broadcast to both receivers, thus each receiver unavoidably gets undesired relay-passing symbols of the other user. Each transmitter provides the other receiver with proper side information that can help resolve the unavoidable collisions. At Time 2, for example, active transmitter 2 provides receiver 1 with b_1 and b_5 as side information when transmitter 1 is inactive, and receiver 1 exploits them to resolve the interference in $a_2 + b_1$ and $a_5 + b_5$. Overall, transmitter 1 always sends four fresh symbols to achieve interference-free DoF, and the symbols arrive at receiver 1, either directly or through the relay, sometimes interfered by relay-passing symbols of user 2. Receiver 1 resolves the interference and decodes its desired symbols with side information provided by transmitter 2.

Remark 5 (Other Examples That Exploit Side Information): The gain obtained by exploiting side information appears in many other network examples. In [20], a wireless router receives multiple packets intended for different destinations from its neighboring nodes, encodes them into one packet, and broadcasts it. In a single transmission, every destination decodes its desired packet by using the packets for the other destinations that it overheard as side information. In [21], channel output feedback can increase the non-feedback capacity of the Gaussian IC. In [22], outdated channel state feedback can increase the non-feedback capacity of the Gaussian MIMO broadcast channel. In both works, feedback enables receivers to exploit their past received signals as side information.

V. Proof of Theorem 2

The bound (1) is the cut-set bound, so we omit the proof. The bound (2) consists of two bounds on d_1+d_2, and we derive them in this section. In our derivations (see (3) and (4)), as shown at the top of the next page, we can get two additional bounds on $R_1 + R_2$ by changing the order of R_1 and R_2. But these bounds on $R_1 + R_2$ result in the identical bounds on $d_1 + d_2$ due to symmetry, so we omit their proofs. The outer bound proof follows the genie-aided approach. For notational convenience, we use $\sum$ to indicate $\sum_{t=1}^{n}$, S_t to indicate (S_{1t}, S_{2t}), and S^n to indicate the sequence of S up to n.

One bound on $R_1 + R_2$ can be derived as in (3): (a) is from Fano's inequality; (b) is from the mutual independence of (W_1, W_2, S^n); (c) is from conditioning reduces entropy; (d) is from $X_{kt} = f_{kt}(W_k, S^{t-1})$ and $X_{Rt} = f_{Rt}(Y_R^{t-1}, S^t)$, the mutual independence of $(Z_1^n, Z_2^n, Z_R^n, W_1, W_2, S^n)$, the i.i.d. assumption of (Z_1^n, Z_2^n, Z_R^n), and conditioning reduces entropy; (e) is from conditioning reduces entropy, and the evaluation of S_t.

The other bound on R_1+R_2 can be derived as in (4): (a) is from Fano's inequality; (b) is from the mutual independence of (W_1, W_2, S^n); (c) is from conditioning reduces entropy; (d) is from $X_{kt} = f_{kt}(W_k, S^{t-1})$ and $X_{Rt} = f_{Rt}(Y_R^{t-1}, S^t)$, the mutual independence of $(Z_1^n, Z_2^n, Z_R^n, W_1, W_2, S^n)$, the i.i.d. assumption of (Z_1^n, Z_2^n), and conditioning reduces entropy; (e) is from conditioning reduces entropy, and the evaluation of S_t.

To get the claimed outer bound on $d_1 + d_2$, we evaluate the above bounds with the Gaussian distributions that maximize the differential entropies [4], and take the limit as $P \to \infty$ after dividing them by $\log(P)$.

VI. Discussion

A. On Optimality of Our Schemes

In this work, we established a necessary condition and a sufficient condition for interference-free DoF performances, but they are not identical. For a class of antenna configurations $(\mathcal{C}_3 - \mathcal{C}_3')$, optimality in terms of DoF was not shown. For a resolved class $(\mathcal{C}_1)$, a simple and naive scheme achieves optimality. Both transmitters send signals and the relay sends nothing. For the other resolved classes ($\mathcal{C}_2$ and $\mathcal{C}_3'$), our scheme relies on zero-forcing precoding of the transmitters, through which the transmitters minimize the reception of undesired symbols at the receiving nodes and resolve unavoidable collisions in cooperation with the relay. For the unresolved class $(\mathcal{C}_3 - \mathcal{C}_3')$, however, it is straightforward to see that each transmitter can no longer apply complete zero-forcing precoding, thus it loses control over the destinations of its transmitted symbols. It becomes more difficult for the transmitters not only to avoid undesirable collisions at the receiving nodes, but also to cooperate with the relay. It is still not clear if we need a better achievable scheme, a tighter outer bound, or both.

B. Model Assumptions

In this paper, for the sake of simplicity in our initial attempts to explore mmWave channels, we focus on a channel model where only two Bernoulli random states that capture signal outages are introduced, which represents one specific practical scenario. However, in a variety of practical scenarios, more than two random states that govern signal outages are needed, since any link between a transmitting node and a receiving node can be prone to signal outages due to, say, blockage and severe path-loss along the signal traversal path.

In this regard, it will be worthwhile to examine such channels in further detail and see whether there exists a scheme in which transmitting nodes (both the transmitters and the relay) can perform advanced cooperation by exploiting information of the bursty channel states. It will also be interesting to investigate K-user bursty ICs and examine the benefits of incorporating relays (whose operational type can be combinations of in-band/out-of-band and reception/transmission) into the channels.

VII. Conclusion

We discovered that an in-band relay can provide a DoF gain in the two-user bursty MIMO Gaussian IC. We demonstrated that the relay can help achieve interference-free DoF performances in certain antenna configurations. The relay and the transmitters cooperate by exploiting information of the bursty channel states to achieve such performances. Moreover, we observed that the gain can be particularly substantial in high-outage circumstances, as it can grow linearly with the number of antennas at the relay. Our results show promising benefits that relays can bring into future wireless networks that seek to exploit mmWave spectrum bands, where transmitted signals are highly prone to outages largely due to significant path-loss and increased directivity.

Appendix A
Proof of Corollary 1

This appendix proves the necessary condition for attaining interference-free DoF in Corollary 1 by examining when (2) becomes inactive.

A. $2M \leq N$

1) Individual DoF Bound: $M < N + L$ gives us

$$d_1, d_2 \leq \min\{p(M), p\min(M+L, N) + (1-p)\min(L, N)\}.$$

From $M \leq M + L$ and $M < N$, we have $p(M) \leq p\min(M + L, N)$. Hence, the individual DoF is bounded by

$$d_1, d_2 \leq pM.$$

$$
\begin{aligned}
&n(R_1+R_2-\epsilon_n)\\
&\quad\overset{(a)}{\le} I(W_1;Y_1^n,S^n)+I(W_2;Y_2^n,S^n)\overset{(b)}{\le} I(W_1;Y_1^n|S^n)+I(W_2;Y_1^n,Y_2^n,Y_R^n|S^n,W_1)\\
&\quad\overset{(c)}{\le} \sum h(Y_{1t}|S^n,Y_1^{t-1})-\sum h(Y_{1t}|S^n,W_1,W_2,Y_1^{t-1},Y_2^{t-1},Y_R^{t-1})\\
&\qquad+\sum h(Y_{2t},Y_{Rt}|S^n,W_1,Y_1^{t-1},Y_2^{t-1},Y_R^{t-1},Y_{1t})-\sum h(Y_{2t},Y_{Rt}|S^n,W_1,W_2,Y_1^{t-1},Y_2^{t-1},Y_R^{t-1},Y_{1t})\\
&\quad\overset{(d)}{\le} \sum h(Y_{1t}|S_t)-\sum h(Z_{1t})+\sum h(Y_{2t},Y_{Rt}|S_t,X_{1t},X_{Rt},Y_{1t})-\sum h(Z_{2t},Z_{Rt})\\
&\quad\overset{(e)}{\le} p^2\sum h\left(\mathbf{H}_{11}X_{1t}+\mathbf{H}_{12}X_{2t}+\mathbf{H}_{1R}X_{Rt}+Z_{1t}\right)+p(1-p)\sum h\left(\mathbf{H}_{11}X_{1t}+\mathbf{H}_{1R}X_{Rt}+Z_{1t}\right)\\
&\qquad+(1-p)p\sum h\left(\mathbf{H}_{12}X_{2t}+\mathbf{H}_{1R}X_{Rt}+Z_{1t}\right)+(1-p)^2\sum h\left(\mathbf{H}_{1R}X_{Rt}+Z_{1t}\right)-\sum h\left(Z_{1t}\right)\\
&\qquad+p\sum h\left(\mathbf{H}_{22}X_{2t}+Z_{2t},\mathbf{H}_{R2}X_{2t}+Z_{Rt}|\mathbf{H}_{12}X_{2t}+Z_{1t}\right)-p\sum h\left(Z_{2t},Z_{Rt}\right).
\end{aligned}\tag{3}
$$

$$
\begin{aligned}
&n(R_1+R_2-\epsilon_n)\\
&\quad\overset{(a)}{\le} I(W_1;Y_1^n,S^n)+I(W_2;Y_2^n,S^n)\overset{(b)}{\le} I(W_1;Y_1^n|S^n)+I(W_2;Y_1^n,Y_2^n|S^n,W_1)\\
&\quad= \sum I(W_1;Y_{1t}|S^n,Y_1^{t-1})+\sum I(W_2;Y_{1t}|S^n,W_1,Y_1^{t-1},Y_2^{t-1})+\sum I(W_2;Y_{2t}|S^n,W_1,Y_1^{t-1},Y_2^{t-1},Y_{1t})\\
&\quad\overset{(c)}{\le} \sum h(Y_{1t}|S^n,Y_1^{t-1})-\sum h(Y_{1t}|S^n,W_1,W_2,Y_1^{t-1},Y_2^{t-1},Y_R^{t-1})\\
&\qquad+\sum h(Y_{2t}|S^n,W_1,Y_1^{t-1},Y_2^{t-1},Y_{1t})-\sum h(Y_{2t}|S^n,W_1,W_2,Y_1^{t-1},Y_2^{t-1},Y_R^{t-1},Y_{1t})\\
&\quad\overset{(d)}{\le} \sum h(Y_{1t}|S_t)-\sum h(Z_{1t})+\sum h(Y_{2t}|S_t,X_{1t},Y_{1t})-\sum h(Z_{2t})\\
&\quad\overset{(e)}{\le} p^2\sum h\left(\mathbf{H}_{11}X_{1t}+\mathbf{H}_{12}X_{2t}+\mathbf{H}_{1R}X_{Rt}+Z_{1t}\right)+p(1-p)\sum h\left(\mathbf{H}_{11}X_{1t}+\mathbf{H}_{1R}X_{Rt}+Z_{1t}\right)\\
&\qquad+(1-p)p\sum h\left(\mathbf{H}_{12}X_{2t}+\mathbf{H}_{1R}X_{Rt}+Z_{1t}\right)+(1-p)^2\sum h\left(\mathbf{H}_{1R}X_{Rt}+Z_{1t}\right)-\sum h\left(Z_{1t}\right)\\
&\qquad+p\sum h\left(\mathbf{H}_{22}X_{2t}+\mathbf{H}_{2R}X_{Rt}+Z_{2t}|\mathbf{H}_{12}X_{2t}+\mathbf{H}_{1R}X_{Rt}+Z_{1t}\right)\\
&\qquad+(1-p)\sum h\left(\mathbf{H}_{2R}X_{Rt}+Z_{2t}|\mathbf{H}_{1R}X_{Rt}+Z_{1t}\right)-\sum h\left(Z_{2t}\right).
\end{aligned}\tag{4}
$$

2) Sum DoF Bound: $M < N$ gives us

$$d_1+d_2 \le p^2\min(2M+L,N)+2p(1-p)\min(M+L,N)+(1-p)^2\min(L,N).$$

From $2M \le 2M+L$ and $2M \le N$, we have $p^2(2M) \le p^2\min(2M+L,N)$. From $M \le M+L$ and $M < N$, we have $2p(1-p)(M) \le 2p(1-p)\min(M+L,N)$. The sum of these two inequalities shows that the sum of the two individual DoF bounds is tighter than the sum DoF bound. Hence, the sum DoF is bounded by

$$d_1+d_2 \le 2pM.$$

Therefore, $2M \le N$ is a necessary condition for attaining interference-free DoF for all $p < 1$.

B. $M \le N < 2M$

1) Individual DoF Bound: $M \le N+L$ gives us

$$d_1,d_2 \le \min\{p(M),\ p\min(M+L,N)+(1-p)\min(L,N)\}.$$

From $M \le M+L$ and $M \le N$, we have $p(M) \le p\min(M+L,N)$. Hence, the individual DoF is bounded by

$$d_1,d_2 \le pM.$$

2) Sum DoF Bound: $M \le N$ and $2M+L > N$ give us

$$d_1+d_2 \le p^2(N)+2p(1-p)\min(M+L,N)+(1-p)^2\min(L,N).$$

If we choose a looser sum DoF bound and show the looser sum DoF bound is strictly tighter than the sum of the two individual DoF bounds, then we show the actual sum DoF bound is also strictly tighter. From $\min(M+L,N) \le N$ and $\min(L,N) \le N$, we have $2p(1-p)\min(M+L,N) \le 2p(1-p)(N)$ and $(1-p)^2\min(L,N) \le (1-p)^2(N)$. Hence, the sum DoF is bounded by

$$d_1+d_2 \le N.$$

For $\frac{N}{2M} < p < 1$, the looser sum DoF bound is strictly tighter than the sum of the two individual DoF bounds.

Therefore, $M \le N < 2M$ is not a necessary condition for attaining interference-free DoF for all $p < 1$.

C. $N < M < 2N$

1) Individual DoF Bound: $M+L > N$ gives us

$$d_1,d_2 \le \min\{p\min(M,N+L),\ p(N)+(1-p)\min(L,N)\}.$$

2) Sum DoF Bound: $M - N < N + L$, $2M + L > N$, and $M + L > N$ give us

$$d_1 + d_2 \le \min \left\{ \begin{array}{l} p(M-N), \\ p\min(M+L-N, N) \\ +(1-p)\min\{(L-N)^+, N\} \end{array} \right\} + p^2(N) + 2p(1-p)(N) + (1-p)^2 \min(L, N).$$

From $M - N \le M + L - N$ and $M - N < N$, we have $p(M-N) \le p\min(M+L-N, N)$. Hence, the sum DoF is bounded by

$$d_1 + d_2 \le p(M-N) + p^2(N) + 2p(1-p)(N) + (1-p)^2 \min(L, N).$$

Given M, N, and L, the difference between the two terms of the big minimum function in the individual DoF bound is a continuous function $f(p)$ in $p \in [0, 1]$. When $L > 0$, by the intermediate value theorem, there exists $p_0 \in [0, 1]$ such that $f(p_0) = 0$ since $f(0)f(1) < 0$. And, $p_0 \in (0, 1)$ since $f(0) \neq 0$ and $f(1) \neq 0$. Hence, the second term is active for $p_0 \le p < 1$. When $L = 0$, the second term is always active.

For $p_0 \le p < 1$, the following inequality should necessarily hold to attain interference-free DoF.

$$\begin{aligned} 2p(N) + 2(1-p)\min(L, N) &\le p(M-N) + p^2(N) + 2p(1-p)(N) \\ &\quad + (1-p)^2 \min(L, N). \end{aligned}$$

When $L \ge N$, the inequality becomes

$$p \ge \frac{N}{M-N}.$$

The above inequality does not hold for $p_0 \le p < 1$.

When $L < N$, the inequality becomes

$$p^2(L-N) + p(M-N) - L \ge 0.$$

From $L < N$, $g(p) = p^2(L-N) + p(M-N) - L$ is a strictly concave quadratic function. $g(1) \ge 0$ should necessarily hold for the above inequality to hold for $p_0 \le p < 1$. But $g(1) = M - 2N < 0$.

Therefore, $N < M < 2N$ is not a necessary condition for attaining interference-free DoF for all $p < 1$.

D. $M \ge 2N$ and $L < N$

1) Individual DoF Bound: $M > N + L$, $M + L > N$, and $L < N$ give us

$$d_1, d_2 \le \min\{p(N+L), p(N) + (1-p)(L)\}.$$

2) Sum DoF Bound: $M + L - N \ge N$, $L < N$, $2M + L > N$, and $M + L > N$ give us

$$d_1 + d_2 \le \min\{p\min(M-N, N+L), p(N)\} + p^2(N) + 2p(1-p)(N) + (1-p)^2(L).$$

From $M - N \ge N$ and $N + L \ge N$, we have $p\min(M-N, N+L) \ge p(N)$. Hence, the sum DoF is bounded by

$$d_1 + d_2 \le p(N) + p^2(N) + 2p(1-p)(N) + (1-p)^2(L).$$

For $p < \frac{1}{2}$, in the individual DoF bound, $p(N+L)$ term is active. Otherwise, $p(N) + (1-p)(L)$ term is active.

For $p < \frac{1}{2}$, the following inequality should necessarily hold to attain interference-free DoF.

$$2p(N+L) \le p(N) + p^2(N) + 2p(1-p)(N) + (1-p)^2(L).$$

The inequality becomes $p(1-p)N \ge (-p^2 + 4p - 1)L$.

For $\frac{1}{2} \le p < 1$, the following inequality should necessarily hold to attain interference-free DoF.

$$\begin{aligned} &2p(N) + 2(1-p)(L) \\ &\quad \le p(N) + p^2(N) + 2p(1-p)(N) + (1-p)^2(L). \end{aligned}$$

The inequality becomes $p \ge \frac{L}{N-L}$.

$3L \le N$ should necessarily hold for both inequalities to hold.

Therefore, $M \ge 2N$ and $3L \le N$ is a necessary condition for attaining interference-free DoF for all $p < 1$.

E. $M \ge 2N$ and $N \le L < 2N$

1) Individual DoF Bound: $M + L > N$ and $L \ge N$ give us

$$d_1, d_2 \le \min\{p\min(M, N+L), N\}.$$

From $M \ge 2N$ and $N + L \ge 2N$, we have $p\min(M, N+L) \ge p(2N)$. Hence, for $\frac{1}{2} \le p < 1$, the second term of the big minimum function is active.

$$d_1, d_2 \le N.$$

2) Sum DoF Bound: $M + L - N > N$, $L - N < N$, $2M + L > N$, $M + L > N$, and $L \ge N$ give us

$$d_1 + d_2 \le \min \left\{ \begin{array}{l} p\min(M-N, N+L), \\ p(N) + (1-p)(L-N) \end{array} \right\} + N.$$

From $\min(a, b) \le a$ and $\min(a, b) \le b$, the sum DoF is bounded by

$$d_1 + d_2 \le p(N) + (1-p)(L-N) + N.$$

From $L - N < N$, we have $p(N) + (1-p)(L-N) < N$. Hence, the sum DoF is strictly tighter than the sum of the two individual DoF bounds for $\frac{1}{2} \le p < 1$.

Therefore, $M \ge 2N$ and $N \le L < 2N$ is not a necessary condition for attaining interference-free DoF for all $p < 1$.

F. $M \ge 2N$ and $L \ge 2N$

1) Individual DoF Bound: $M + L > N$ and $L > N$ give us

$$d_1, d_2 \le \min\{p\min(M, N+L), N\}.$$

2) Sum DoF Bound: $M + L - N > N$, $L - N \ge N$, $2M + L > N$, $M + L > N$, and $L > N$ give us

$$d_1 + d_2 \le \min\{p\min(M-N, N+L), N\} + N.$$

Let p_1^* and p_2^* be the threshold probabilities that activate the second terms of the first and second big minimum functions, respectively.

$$p_1^* = \frac{N}{\min(M, N+L)}, \quad p_2^* = \frac{N}{\min(M-N, N+L)}.$$

When $M - N < N + L$, $p_1^* < p_2^*$ holds. For $p_1^* \leq p < p_2^*$, the individual DoF is bounded by N, and the sum DoF is bounded by $p(M - N) + N$. From $p < p_2^* = \frac{N}{M-N}$, we have $p(M - N) < N$. Hence, the sum DoF bound is strictly tighter than the sum of the two individual DoF bounds.

When $M - N \geq N + L$, the individual DoF is bounded by $\min\{p(N + L), N\}$, and the sum DoF is bounded by $\min\{p(N + L), N\} + N$. Hence, the sum of the two individual DoF bounds is tighter than the sum DoF bound.

Therefore, $M \geq 2N + L$ and $L \geq 2N$ is a necessary condition for attaining interference-free DoF for all $p < 1$, whereas $2N \leq M < 2N + L$ and $L \geq 2N$ is not.

The necessary condition for attaining interference-free DoF in Corollary 1 is proved.

Appendix B
Generalization of Section IV-B

All transmitting nodes always apply zero-forcing precoding. Each transmitter uses $2N + L$ antennas. It sends N fresh symbols to its intended receiver, and L fresh symbols to the relay. It uses the remaining N antennas to participate in cooperative interference nulling. The relay uses L antennas when receiving, and $2N$ antennas when sending. It uses N antennas for receiver 1 only, and the other N antennas for receiver 2 only.

Except for the number of antennas being used, there is little difference between the generalized scheme and the scheme presented in Section IV-B. Since the relay is shared by both users, unavoidable collisions take place at the relay. A key idea is that the relay cooperates with active transmitters to remove the interference in the air and deliver only desired symbols to each receiver.

Until both transmitters become active, the channel can be viewed as two independent bursty relay channels.

When both transmitters become active, L collisions occur at the relay. From this point, each transmitter starts to send N symbols to the other receiver to participate in cooperative interference nulling. From feedback, the transmitters are aware of the past collisions and the order of the occurrences. In resolving the past collisions, they send N symbols to the other receiver in a first-in-first-out (FIFO) manner. Also from feedback, the transmitters figure out whether or not they have succeeded in cooperative interference nulling. The relay sends its symbols adaptively for cooperative interference nulling based on current channel states.

There is one special case to note: a series of successive occurrences of both transmitters being active. After the first occurrence, both transmitters start to send symbols to the other receiver to resolve the past collisions at the relay. This would cause interference at the receivers if both transmitters become active again. But the relay can prevent such interference from occuring. Let us consider an example of two successive occurrences of both transmitter being active.

- Time 1: Transmitter 1 sends $(a_1, \cdots, a_{N+L})$, transmitter 2 sends $(b_1, \cdots, b_{N+L})$, and the relay sends nothing. L collisions $(a_1 + b_1, \cdots, a_L + b_L)$ occur at the shared relay. $(a_{L+1}, \cdots, a_{N+L})$ and $(b_{L+1}, \cdots, b_{N+L})$ arrive at the intended receivers without interference.
- Time 2: To participate in cooperative interference nulling, transmitter 1 sends $(-a_1, \cdots, -a_N)$ to receiver 2, in addition to a new set of fresh symbols $(a'_1, \cdots, a'_{N+L})$ toward receiver 1 and the relay. Also, transmitter 2 sends $(-b_1, \cdots, -b_N)$ to receiver 1, and $(b'_1, \cdots, b'_{N+L})$ toward receiver 2 and the relay. This would cause interference: $(a'_{L+1} - b_1, \cdots, a'_{N+L} - b_N)$ at receiver 1, and $(b'_{L+1} - a_1, \cdots, b'_{N+L} - a_N)$ at receiver 2. But the relay knows the current channel states, so it sends $(a_1 + b_1, \cdots, a_L + b_L)$ to both receivers. As a result, there is no interference at both receivers: $(a'_{L+1} + a_1, \cdots, a'_{N+L} + a_N)$ at receiver 1, and $(b'_{L+1} + b_1, \cdots, b'_{N+L} + b_N)$ at receiver 2. Since all desired relay-passing symbols will be eventually decoded, the new set of N fresh symbols at each receiver will be also decoded.

At each receiver, there is no interference at all times. The relay exploits information of current channel states, and always makes sure there is no interference at both receivers in cooperation with active transmitters. Overall, each transmitter-receiver link communicates as if there is no other link nearby.

An analysis similar to that in Section IV-B is as follows.

A. High-Outage Regime $p < \frac{N}{N+L}$

- Type 1 symbols: collisions. L symbols are reserved at the relay with probability p^2, and N symbols can be delivered to receiver 1 through cooperation between the relay and transmitter 2 with probability $(1 - p)p$.

$$p^2 \times L < (1 - p)p \times N.$$

- Type 2 symbols: collision-free. L symbols are reserved at the relay with probability $p(1 - p)$, and N symbols can be delivered to receiver 1 by the relay with probability $(1 - p)^2$.

$$p(1 - p) \times L < (1 - p)^2 \times N.$$

In the high-outage regime, the above conditions hold. Each transmitter sends $N + L$ fresh symbols at the rate of p due to outages, and all of them will be eventually decoded at the intended receiver. This achieves the individual DoF of $p(N + L)$.

B. Low-Outage Regime $\frac{N}{N+L} \leq p < 1$

In this regime, both transmitters send symbols at a *lower* rate. This lowering is represented as a multiplicative factor q.

- Type 1 symbols: $(pq)^2 \times L < (1 - pq)(pq) \times N$.
- Type 2 symbols: $(pq)(1 - pq) \times L < (1 - pq)^2 \times N$.

In the low-outage regime, defining q as $\frac{1}{p}(\frac{N}{N+L} - \epsilon)$, where $\epsilon > 0$, satisfies the above conditions. Each transmitter sends $N + L$ fresh symbols at the rate of pq, and all of them will be eventually decoded at the intended receiver. This achieves the individual DoF of $pq(N + L)$. As both transmitters choose ϵ arbitrarily close to zero, the individual DoF converges to N.

$$pq(N + L) = \left(\frac{N}{N + L} - \epsilon\right)(N + L) \to N.$$

The proposed scheme achieves the following DoF region.

$$\mathcal{D} = \{(d_1, d_2) : d_1, d_2 \leq \min(p(N + L), N)\}.$$

APPENDIX C GENERALIZATION OF SECTION IV-C

Both transmitters always apply zero-forcing precoding, but the relay cannot. Each transmitter uses $2N + L$ antennas. It sends N fresh symbols to its intended receiver, and L fresh symbols to the relay. It uses the remaining N antennas to provide the other receiver with side information. The relay uses L antennas when receiving and sending.

Except for the number of antennas being used, there is little difference between the generalized scheme and the scheme presented in Section IV-C. Since the relay broadcasts its symbols due to its limited number of antennas, unavoidable collisions take place at the receivers. A key idea is that each transmitter provides the other receiver with side information to help the receiver resolve the interference.

Each transmitter provides the other receiver with side information at all times. It provides the duplicate of its relay-passing symbols, at most $N - L$ of them, in a first-in-first-out (FIFO) manner. From feedback, the transmitters figure out whether or not they have succeeded in providing side information. Each receiver has N antennas. The receiver uses L antennas to get symbols from the relay when its corresponding transmitter is inactive. And, the receiver uses $N - L$ antennas to get side information from the other transmitter if it is active.

The relay performs its receive-broadcast operation in a FIFO manner. When there is an inactive transmitter, the relay broadcasts the oldest L symbols that have not been delivered to the corresponding receiver. This causes interference at the other receiver. When both transmitters are inactive, the relay broadcasts the L sums of the oldest L symbols of both users, so that both receivers get useful symbols.

An analysis similar to that in Section IV-C is as follows.

A. High-Outage Regime $p < \frac{1}{2}$

- User 1's relay-passing symbols: L symbols are reserved with probability p, and L symbols can be delivered to receiver 1 by the relay with probability $1 - p$.

$$p \times L < (1 - p) \times L.$$

- User 2's relay-passing symbols: L symbols are reserved with probability p, and eventually broadcast. Receiver 1 can get the duplicate of $N - L$ user 2's relay-passing symbols as side information from transmitter 2 with probability $(1 - p)p$.

$$p \times L < (1 - p)p \times (N - L).$$

In the high-outage regime, the above conditions hold. Each transmitter sends $N + L$ fresh symbols at the rate of p due to outages, and all of them will be eventually decoded at the intended receiver in the high-outage regime. This achieves the individual DoF of $p(N + L)$.

B. Low-Outage Regime $\frac{1}{2} \leq p < 1$

In this regime, both transmitters send symbols to the relay at a *lower* rate. This lowering is represented as a multiplicative factor q.

- User 1's relay-passing symbols are reserved at the rate of $pq \times L$, and can be delivered at the rate of $(1 - p) \times L$.

$$pq \times L < (1 - p) \times L.$$

- User 2's relay-passing symbols are reserved at the rate of $pq \times L$, and eventually broadcast. Receiver 1 can get side information at the rate of $(1 - p)p \times (N - L)$.

$$pq \times L < (1 - p)p \times (N - L).$$

In the low-outage regime, defining q as $\frac{1}{p}(1-p-\epsilon)$, where $\epsilon > 0$, satisfies the above conditions. Each transmitter sends N fresh symbols to its intended receiver at the rate of p, and L fresh symbols to the relay at the rate of pq. All of them will be eventually decoded at the intended receiver. This achieves the individual DoF of $pN + pqL$. As both transmitters choose ϵ arbitrarily close to zero, the individual DoF converges to $pN + (1 - p)L$.

$$pN + pqL = pN + (1 - p - \epsilon)L \rightarrow pN + (1 - p)L.$$

The proposed scheme achieves the following DoF region.

$$\mathcal{D} = \{(d_1, d_2) : d_1, d_2 \leq pN + \min(p, 1 - p)L\}.$$

APPENDIX D PROOF OF THEOREM 4

This appendix proves Theorem 4. Theorem 2 proves the converse. The cut-set argument proves achievability of the individual DoF [4]. We prove achievability of the sum DoF.

A. High-Outage Regime $p < \frac{1}{2}$

The transmitters always send fresh symbols. The relay operates as follows.

When only one transmitter is active, the relay sends nothing. The intended receiver decodes its desired symbol.

When both transmitters are active, the relay sends nothing. Each receiver cannot decode its desired symbol instantaneously. It gets a linear sum of its desired symbol and an undesired symbol from the other transmitter. The relay gets another linear sum that is linearly independent of the linear sum at each receiver.

When both transmitters become inactive, the relay forwards the linear sum. Then, each receiver can decode its desired symbol that it could not decode due to interference.

In summary, both receivers get a collision of their desired symbol and an undesired symbol of the other transmitter when both transmitters are active. Both receivers decode their desired symbol in the collision when they get an extra linear sum from the relay when both transmitters become inactive.

In the high-outage regime, it is more likely for both transmitters to be inactive ($(1 - p)^2$) than active (p^2). In other words, the relay can forward extra linear sums to help the receivers resolve collisions more often than both receivers get collisions. Decoding of all desired symbols is guaranteed at both receivers. The sum DoF is $2p$.

B. Low-Outage Regime $\frac{1}{2} \leq p < 1$

The transmitters send fresh symbols at a *lower* rate to guarantee decoding. At any time instant, each transmitter chooses to send a symbol with probability q. Each transmitter makes such decisions independently over time, and the decisions of the transmitters are independent. Hence, each transmitter is in fact active with probability pq.

In the low-outage regime, both transmitters set q to be $\frac{1-\epsilon}{2p}$. Then, it is more likely for both transmitters to be inactive ($(1-pq)^2$) than active ($(pq)^2$). Decoding of all desired symbols is guaranteed at both receivers. The sum DoF is $2pq$. As both transmitters choose ϵ arbitrarily close to zero, the sum DoF converges to 1.

The proposed scheme achieves the following DoF region.

$$\mathcal{D} = \{(d_1, d_2) : d_1, d_2 \leq p,\ d_1 + d_2 \leq \min(2p, 1)\}.$$

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