# Role of a Relay in Bursty Networks with Correlated Transmissions

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***Abstract*—We explore the role of a relay in multiuser networks where some physical perturbation shared around the users may generate data traffic for them simultaneously, hence cause their transmission patterns to be correlated. We investigate how the gain from the help of a relay varies with correlations across the users' transmission patterns in a bursty multiple access channel where the users send signals intermittently. As our main results, we show that in most cases a relay can provide a greater degrees-of-freedom (DoF) gain when the users' transmission patterns are more correlated. Furthermore, we demonstrate that the DoF gain can scale with the number of users.**

## I. Introduction

Relays have been considered unable to provide DoF gains in standard information-theoretic channels where transmitters are usually assumed to send signals at all times [1]. But recent studies have found that relays can provide DoF gains in *bursty* channels where transmitters send signals intermittently [2], [3]. Further findings therein shed new light on the significant role of relays in various bursty channels, making it look promising to employ relays in wireless networks.

The main results of prior work [2] well emphasize practical advantages of employing a relay in bursty multiple access channels (MACs). One is to improve performance in emerging networks, such as device-to-device systems in which mobile devices directly exchange data with little help of base stations. When multiple devices convey bursts of data to a device in such systems, as shown in [2], the assistance of a relay can be useful to achieve higher data throughput, as the relay can provide DoF gains scalable with extra relay antennas, and to enable collision-free communication. Another advantage is to simplify random access protocols to reduce control signaling overhead. When multiple users wish to deliver data to a common destination, some protocols are needed to manage signal collisions for reliable data delivery. It has been demonstrated in [2] that a relay can take a burden off the users when it comes to coping with such collisions. It turns out that the relay can resolve the collisions on behalf of the users. Hence, the users are allowed to send signals at random intervals without extra effort such as retransmissions of collided signals.

In this work, we set out to explore the practical aspects of employing a relay in detail. To that end, we look into bursty MACs where dependencies across the users' intermittent data availabilities cause *correlated transmissions* across the users to occur. Consider a sensor network in which multiple sensors spread gather temperature measurements and convey them to a central hub which computes the average. Alternatively, consider a safety network in which nearby vehicles equipped with sensors detect a possible risk and share the information to prevent the accident to happen. In both, some physical perturbation around objects in close proximity may cause the objects to collect and exchange bursts of data simultaneously. A natural question that arises in this context is: will employing a relay be still useful when multiple users tend to send signals simultaneously, and causing severe collisions?

To answer the question, we consider two extreme yet representative cases. In one case, the users' data availabilities are fully dependent; hence all users send signals simultaneously. In the other case, the users' data availabilities are independent; hence a user sends signals regardless of the others[1]. For each case, we measure the gain from the assistance of a relay. And we compare the two gains to see when employing a relay can be more beneficial.

As our main contribution, we provide insight into how the benefit of employing a relay varies according to correlations across the users' bursty transmission patterns. The most interesting findings are perhaps those in the case where the relay has sufficiently many antennas to achieve the maximum DoF:

- The gain from the assistance of a relay is greater when transmission patterns across the users are more correlated.
- The gain can scale with the number of users.

We note, however, that observations in other cases where the relay does not have enough antennas suggest that employing a relay could sometimes be more beneficial when transmission patterns across the users are less correlated. The antenna settings in such cases represent scenarios in which the relay has very limited numbers of antennas, so it fail to assist well either the transmitters or the receiver. More technical details and discussions to follow will make our findings clear.

**Related work:** Among many studies on relay networks, the most related are [4] and [5]. We obtain our main results by extending noisy network coding for multimessage multicast networks [4] in which relays use compress-forward strategies [5]. To the best of our knowledge, there has been little work done on multiuser networks where correlations across the users' transmission patterns are taken into account.

[1]Note that it is "the availabilities of data at a given time" that may be correlated across the users. The messages of the users are assumed independent. In this work, by any terms regarding "correlations" or "dependencies", we mean the data availabilities across the users, not the messages.

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## II. MODEL

Fig. 1 describes the $K$-user bursty MIMO Gaussian MAC with a relay. The transmitters, receiver and relay have $M$, $N$ and $L$ antennas, respectively. Transmitter $k$ wishes to reliably deliver a message $W_k$ to the receiver, $\forall k = 1, 2, \ldots, K$.

Let $X_{kt} \in \mathbb{C}^M$ be transmitter $k$'s encoded signal and $X_{Rt} \in \mathbb{C}^L$ be the relay's encoded signal at time $t$. The encoded signals are power-constrained: $\mathbb{E}[|X_{kt}^2|] \leq P$ and $\mathbb{E}[|X_{Rt}^2|] \leq P$. Traffic states $S_{kt}$ are assumed to follow $\mathsf{Bern}(p)$ to govern bursty transmissions over time. Joint distribution[2] $\mathsf{P}(S_1, S_2, \ldots, S_k)$ captures the bursty transmission patterns of the users. Unlike the transmitters, the relay is not subject to bursty transmissions.

Let us note the rationale behind our modeling. This work is motivated by sensor networks where transmitters are simple devices, thus less capable; they can neither have a large buffer to store ample data, nor employ advanced scheduling. Intermittent data traffic forces the users to send signals whenever they have available data, leading to bursty behaviors. In contrast, a relay can be equipped with rich capabilities such as a large buffer and channel state sensing; it can store sufficient past received signals and use them later according to channel states for better assistance. Hence, unlike the transmitters, the relay can send signals at all times. As we consider intermittent data traffic to be a cause of bursty transmissions, it might be more practically relevant to model such burstiness in higher layers of the communication protocol stack. However, to simplify the model greatly while capturing the bursty nature to some extent, we incorporate random states into the physical channel.

Additive noise terms $Z_t$ at the receiver and $Z_{Rt}$ at the relay are assumed to be independent, distributed according to $\mathcal{CN}(0, \mathbf{I}_N)$ and $\mathcal{CN}(0, \mathbf{I}_L)$, and i.i.d. over time. Let $Y_t \in \mathbb{C}^N$ be the received signal at the receiver and $Y_{Rt} \in \mathbb{C}^L$ be the received signal at the relay at time $t$.

$$Y_t = \sum_k \mathbf{H}_k S_{kt} X_{kt} + \mathbf{H}_R X_{Rt} + Z_t,$$
$$Y_{Rt} = \sum_k \mathbf{H}_{Rk} S_{kt} X_{kt} + Z_{Rt}.$$

The matrices $\mathbf{H}_k$ and $\mathbf{H}_{Rk}$ describe the time-invariant channels from transmitter $k$ to the receiver and to the relay, respectively. The matrix $\mathbf{H}_R$ describes the time-invariant channel from the relay to the receiver. All channel matrices are assumed to be full-rank.

We assume current traffic states are available at the receiver and the relay. Each transmitter knows its current traffic state, as it has access to the availability of data for transmission.

Transmitter $k$ encodes its signal at time $t$ based on its own message and its own current traffic state; we assume uncoordinated traffic states across all transmitters, which means each transmitter has access to its own traffic state only. The relay encodes its signal at time $t$ based on its past received signals, and both past and current traffic states of all transmitters.

[2]We let joint distributions capture the users' transmission patterns that stem from correlations across the data availabilities, while for simplicity we assume marginal distributions to be identical.

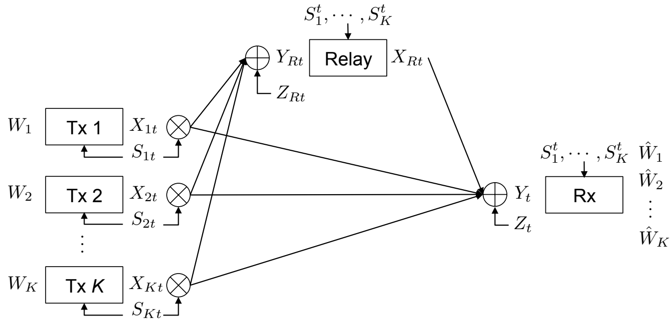



Fig. 1. $K$-user bursty MIMO Gaussian MAC with a relay.

Finally, we define the DoF region $\mathcal{D} := \{(d_1, d_2, \ldots, d_K) : \exists (R_1, R_2, \ldots, R_K) \in \mathcal{C}(P) \text{ and } d_k = \lim_{P\to\infty} \frac{R_k}{\log P}\}$, where $\mathcal{C}(P)$ is the capacity region with power constraint $P$.

## III. MAIN RESULTS

For concreteness, we first state a theorem of prior work [2] that we use to obtain our results of this work. The theorem is obtained under the assumption that the traffic states follow $\mathsf{Bern}(p)$ over time and are independent across the users.

*Theorem 1:* The DoF region of the $K$-user bursty MIMO Gaussian MAC with a relay is characterized as follows.

$$\sum_{k\in\mathcal{S}} d_k \leq \min \begin{bmatrix} \sum_{i=0}^{|\mathcal{S}|} \mathsf{B}_{|\mathcal{S}|}(i) \min(iM, N+L), \\ \sum_{i=0}^{|\mathcal{S}|} \mathsf{B}_{|\mathcal{S}|}(i) \min(iM+L, N) \end{bmatrix},$$

where $\mathcal{S} \subseteq \{1, \ldots, K\}$ and $\mathsf{B}_{|\mathcal{S}|}(i) := \binom{|\mathcal{S}|}{i} p^i (1-p)^{|\mathcal{S}|-i}$.

*Proof:* We give a sketch of the proof. See [2] for details. Extending noisy network coding for multimessage multicast networks [4] can achieve the cut-set bound. The relay takes a receive-forward strategy. The distribution of the traffic states, reflected in $\mathsf{B}_{|\mathcal{S}|}(i)$, specifies how often a certain number of transmitters are active, which determines the number of symbols that can be conveyed from them to the receiver. ■

In this paper, we focus on additive sum DoF gains, and the differences between the sum DoF with a relay and that without a relay, to investigate the number of additional DoF obtained with the assistance of a relay. We now state our main theorem.

*Theorem 2:* The additive sum DoF gain obtained by adding a relay in the $K$-user bursty MIMO Gaussian MAC is

$$\Delta\mathsf{DoF} = \min \begin{bmatrix} \sum_{\mathcal{A}\in\Omega} \mathsf{P}(\mathcal{A}) \min(|\mathcal{A}|M, N+L), \\ \sum_{\mathcal{A}\in\Omega} \mathsf{P}(\mathcal{A}) \min(|\mathcal{A}|M+L, N) \end{bmatrix} - \sum_{\mathcal{A}\in\Omega} \mathsf{P}(\mathcal{A}) \min(|\mathcal{A}|M, N),$$

where $\Omega$ is the set that includes all subsets of $\{1, 2, \ldots, K\}$, $\mathcal{A}$ is a set that indicates which transmitters are active, and $\mathsf{P}(\mathcal{A})$ is a joint distribution that describes the probabilities of the transmitters indicated by $\mathcal{A}$ being active.

*Proof:* See Section V-A. ■

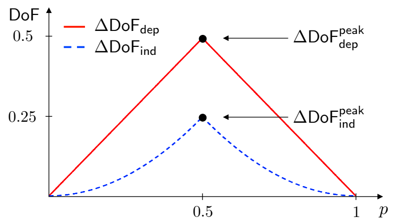



Fig. 2. A relay provides greater DoF gains when the users' data availabilities are dependent. Antenna setting: $(K, M, N, L) = (2, 1, 1, 1)$.

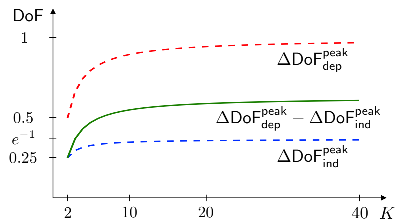



Fig. 3. The growing peak DoF gains with both dependent and independent data availabilities, and the expanding gap between them. Antenna settings: $M = N = 1$ and $L \geq K - 1$.

Although it might be comprehensive to explore how gains from employing a relay change with varying levels of correlations across the users' data availabilities, we consider two extreme ends of the spectrum for simplicity. On the one end, the users' data are available in a fully dependent manner; thus all transmitters are either active or inactive at the same time. On the other end, they are available in an independent manner; thus a transmitter is active regardless of the others.

*Corollary 1:* The additive sum DoF gains obtained by adding a relay in the $K$-user bursty MIMO Gaussian MAC with fully dependent and independent data availabilities are

$$\Delta\mathsf{DoF}_{\mathsf{dep}} = \min \begin{bmatrix} p \min(KM - N, L), \\ (1-p)\min(L, N) \end{bmatrix}, \quad (1)$$

$$\Delta\mathsf{DoF}_{\mathsf{ind}} = \min \begin{bmatrix} \sum_{i=\lfloor \frac{N}{M} \rfloor + 1}^{K} \mathsf{B}_K(i) \min(iM - N, L), \\ \sum_{i=0}^{\lfloor \frac{N}{M} \rfloor} \mathsf{B}_K(i) \min(L, N - iM) \end{bmatrix}. \quad (2)$$

*Proof:* See Section V-A. ∎

## IV. Relay Gains

The role of a relay and its functionality may vary depending on the antenna settings. In this section, we examine three cases.

### A. $L \geq \max(KM - N, N)$: *Relays with enough antennas*

The condition implies that the relay can get additional signals that the receiver needs to resolve the worst-case collisions that occur when all transmitters are active, and also forward the maximum number of signals that the receiver can get at a time. For this case, the relay can help achieving the maximum DoF for a given bursty MAC: $\min(pKM, N)$, that is collision-free DoF in the low traffic regime and saturated DoF in the high traffic regime. Intuitively, the relay receives additional signals when too many transmitters are active, and later forwards them when only a few are active, to achieve the maximum DoF. We have a few interesting findings, illustrated in Figs. 2 and 3.

The first finding is that the gain obtained from the assistance of a relay is greater with dependent data availabilities than that with independent data availabilities:

$$\Delta\mathsf{DoF}_{\mathsf{dep}} > \Delta\mathsf{DoF}_{\mathsf{ind}}, \; p \in (0, 1).$$

See Section V-B for the proof. For this case, in the presence of a relay, the same sum DoF $\min(pKM, N)$ can be achieved regardless of dependencies across the users' data availabilities. Let us see what happens in its absence. With dependent data availabilities, too many symbols are sent simultaneously compared to the number of symbols that the receiver can get at a time. Without a relay, there would be a big DoF loss due to the severe collisions. With independent data availabilities, however, such severe collisions occur less often given the same level of data traffic. Hence, there would be a relatively smaller DoF loss. The absence of a relay costs bursty MACs with dependent data availabilities more, that is, its presence is more beneficial, offering greater DoF gains.

The other finding is regarding the variation of the relay gains with respect to the number of users. To see this variation, and for the sake of simplicity, we assume $M = N = 1$. Also, we focus on two specific values of the relay gains:

$$\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{dep}} := \max_p \Delta\mathsf{DoF}_{\mathsf{dep}}, \;\; \Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{ind}} := \max_p \Delta\mathsf{DoF}_{\mathsf{ind}}.$$

We justify the use of these peak values as a fair means of comparing the relay gains for two reasons. As previously shown, the relay gain is greater with dependent data availabilities for all $p$. Moreover, as will be shown in Section V-C, both relay gains are maximized at the same value of $p^* = \frac{N}{KM}$[3].

We obtain the peak relay gains as a function of $K$ under the assumption $M = N = 1$:

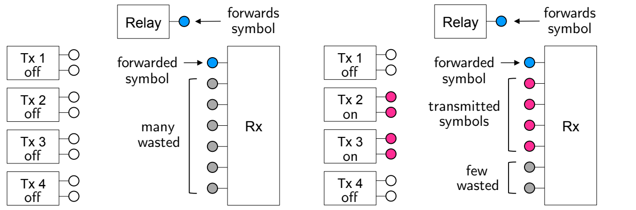
$$\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{dep}} = 1 - \frac{1}{K}, \;\; \Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{ind}} = \left(1 - \frac{1}{K}\right)^K.$$

One can easily verify that both peak gains grow as $K$ increases, and converge to 1 and $\frac{1}{e}$, respectively. This finding shows that the beneficial role of a relay does not deteriorate as the number of users increases. To the contrary, the presence of a relay is more advantageous with more users regardless of dependencies across the users' data availabilities.

More importantly, it turns out that the difference between both peak gains ($\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{dep}} - \Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{ind}}$) grows as $K$ increases. This finding shows that the difference in the amounts of additional DoF attained with the aid of a relay expands as the number of users increases. Considering all findings, we can conclude that employing a relay in bursty MACs is more beneficial with dependencies across the users' data availabilities, and more favorable when more users are involved.

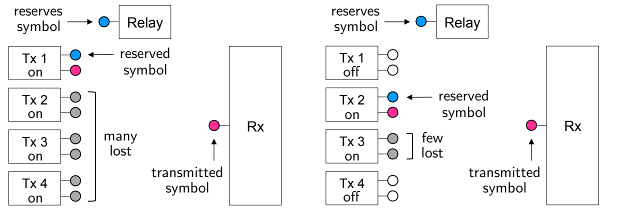
We note that the previous findings are valid for the case where the relay has sufficiently many antennas to help achieving the maximum DoF. In the rest of this section, we examine

[3] We assume $KM > N$. Otherwise, there is no point in discussing relay gains, as the receiver is able to decode all sent symbols instantaneously.



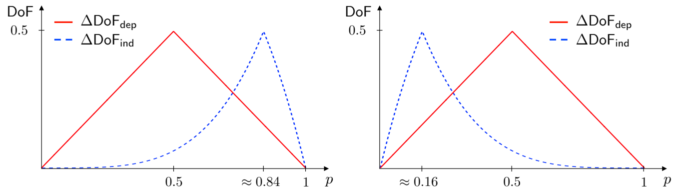

(a) correlated transmissions (b) uncorrelated transmissions

Fig. 4. Antenna setting $(K, M, N, L) = (4, 2, 7, 1)$: $KM - N \leq L \ll N$. Limited capability of the relay in transmission mode stands out with dependent user data availabilities. See Fig. 6(a).




(a) correlated transmissions (b) uncorrelated transmissions

Fig. 5. Antenna setting $(K, M, N, L) = (4, 2, 1, 1)$: $N \leq L \ll KM - N$. Limited capability of the relay in reception mode stands out with dependent user data availabilities. See Fig. 6(b).




(a) $(K, M, N, L) = (4, 2, 7, 1)$ (b) $(K, M, N, L) = (4, 2, 1, 1)$

Fig. 6. DoF gains with independent user data availabilities are greater than those with dependent user data availabilities: when $KM - N \leq L < N$, with high traffic (left) and when $N \leq L < KM - N$, with low traffic (right).

other cases where the relay does not have enough number of antennas to help achieving the maximum DoF, and hence offers limited DoF gains.

### *B. $KM - N \leq L < N$*

A relay offers DoF gains operating in two different modes. It receives additional symbols in reception mode, and forwards them in transmission mode. For this case, particularly when $L \ll N$, the relay may reveal its drawback in transmission mode. In this mode, the relay forwards additional symbols to the receiver, to provide DoF gains by utilizing the receive antennas otherwise unused. In that sense, $L \ll N$ indicates its limited capability to utilize all receive antennas.

The limitation stands out with dependent data availabilities, and makes the relay less beneficial with such dependencies. With dependent data availabilities, all transmitters are either active or inactive. When they are inactive, the relay operates in transmission mode to provide a DoF gain. However, the relay can utilize a very small fraction of receive antennas due to its drawback $L \ll N$, leaving a large fraction of them wasted (Fig. 4(a)). With independent data availabilities, on the other hand, these undesired incidents do not occur as often, given the same data traffic level (Fig. 4(b)).

Fig. 6(a) illustrates the relay providing greater DoF gains with independent data availabilities in high $p$ regimes. This is because in those regimes, the relay is guaranteed to receive enough additional symbols from the transmitters, hence the number of additional symbols it can forward in transmission mode is what determines the amount of DoF gain it offers. In short, the limitation in transmission mode ($L \ll N$), which appears in high $p$ regimes, affects the relay's capability more adversely with dependent data availabilities, hence makes the relay less beneficial with correlated user transmissions.

### *C. $N \leq L < KM - N$*

For this case, particularly when $L \ll KM - N$, the relay may reveal its drawback in reception mode. In this mode, the relay receives and reserves the additional symbols from active transmitters otherwise lost, to provide DoF gains by forwarding them later to the receiver. In that sense, $L \ll KM - N$ indicates its limited capability to reserve all surplus symbols.

The limitation stands out with dependent data availabilities, and makes the relay less beneficial with such dependencies. With dependent data availabilities, when all transmitters are active, the relay operates in reception mode to provide a DoF gain. However, the relay can reserve a very small fraction of surplus symbols due to its drawback $L \ll KM - N$, leaving a large fraction of them lost (Fig. 5(a)). With independent data availabilities, on the other hand, these undesired incidents do not occur as often, given the same data traffic level (Fig. 5(b)).

Fig. 6(b) illustrates the relay providing greater DoF gains with independent data availabilities in low $p$ regimes. This is because in those regimes, the relay is guaranteed to find enough idle moments of the transmitters to forward additional symbols to the receiver, hence the number of additional symbols it can receive in reception mode is what determines the amount of DoF gain it offers. In short, the limitation in reception mode ($L \ll KM - N$), which appears in low $p$ regimes, affects the relay's capability more adversely with dependent data availabilities, hence makes the relay less beneficial with correlated user transmissions.

We have not examined the case $L < \min(KM - N, N)$. The condition implies that the relay's limited capabilities may be revealed in either reception or transmission mode, leading to its worse performance with dependent data availabilities. Regimes in which it underperforms depend on the antenna settings: if $L \ll N$ then it has limitations in transmission mode hence underperforms in high $p$ regimes, and if $L \ll KM - N$ then in low $p$ regimes.

## V. Proofs

### *A. Proof of Theorem 2 and Corollary 1*

Proof of Theorem 2: As briefly noted in the proof sketch of Theorem 1, we extend noisy network coding for multimessage multicast networks [4] to achieve the cut-set bound. Unlike Theorem 1, to consider the case where dependencies across the users' transmissions exist, we replace the distribution $\mathsf{B}_{|\mathcal{S}|}(i)$ in Theorem 1 which represents the users' mutually independent transmissions by the distribution $\mathsf{P}(\mathcal{A})$ in Theorem 2 which can describe the users' dependent transmissions as well.

Proof of Corollary 1: To get $\Delta\mathsf{DoF}_{\mathsf{dep}}$, we set the distribution $\mathsf{P}(\mathcal{A})$ in Theorem 2 as $p$ if $\mathcal{A} = \Omega$ and $1-p$ if $\mathcal{A} = \varnothing$. By some computation, for $KM \le N$ we get $\Delta\mathsf{DoF}_{\mathsf{dep}} = 0$, and for $KM > N$ we get the claimed gain (1). To get $\Delta\mathsf{DoF}_{\mathsf{ind}}$, we set the distribution $\mathsf{P}(\mathcal{A})$ in Theorem 2 as $\mathsf{B}_K(i) = \binom{K}{i} p^i (1-p)^{K-i}$. By some computation, we get the claimed gain (2).

### B. *Proof of* $\Delta\mathsf{DoF}_{\mathsf{dep}} - \Delta\mathsf{DoF}_{\mathsf{ind}} > 0$

With $L \ge KM - N$ and $L \ge N$, from Corollary 1 and some manipulation, we get

$$\Delta\mathsf{DoF}_{\mathsf{dep}} - \Delta\mathsf{DoF}_{\mathsf{ind}} = -pN + \sum_{i=0}^{K} \mathsf{B}_K(i) \min(iM, N).$$

We can verify that $\Delta\mathsf{DoF}_{\mathsf{dep}} - \Delta\mathsf{DoF}_{\mathsf{ind}} > 0$ by showing the second term is always greater than $pN$:

$$\begin{aligned}\sum_{i=0}^{K} \mathsf{B}_K(i) \min(iM, N) &= \sum_{i=1}^{K} \mathsf{B}_K(i) \min(iM, N) \\ &= p \sum_{j=0}^{K-1} \mathsf{B}_{K-1}(j) \min\left(KM, \frac{KN}{j+1}\right) > pN.\end{aligned}$$

The second equality holds since $\mathsf{B}_K(i) = \frac{pK}{i} \mathsf{B}_{K-1}(i-1)$ and by the change of variables $j = i - 1$. The inequality holds since $KM > N$ and $\frac{KN}{j+1} > N$ for $0 \le j < K-1$.

### C. *Proof of increasing* $\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{dep}}$, $\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{ind}}$

With $L \ge KM - N$ and $L \ge N$, from Corollary 1 and some manipulation, we get

$$\begin{aligned}\Delta\mathsf{DoF}_{\mathsf{dep}} &= \min\big(p(KM-N), (1-p)N\big), \\ \Delta\mathsf{DoF}_{\mathsf{ind}} &= \min(pKM, N) - \sum_{i=0}^{K} \mathsf{B}_K(i) \min(iM, N).\end{aligned}$$

It is straightforward to verify that $\Delta\mathsf{DoF}_{\mathsf{dep}}$ is maximized at $p^* = \frac{N}{KM}$. To verify that $\Delta\mathsf{DoF}_{\mathsf{ind}}$ is also maximized at $p^*$, we consider two cases: $0 < p < \frac{N}{KM}$ and $\frac{N}{KM} \le p < 1$. For $0 < p < \frac{N}{KM}$, we get

$$\Delta\mathsf{DoF}_{\mathsf{ind}} = \sum_{i=\lfloor \frac{N}{M} \rfloor + 1}^{K} \mathsf{B}_K(i)(iM - N),$$

where the equality holds since $\sum_{i=0}^{K} \mathsf{B}_K(i) \cdot iM = pKM$. By taking the derivative of the equality, we can verify that $\Delta\mathsf{DoF}_{\mathsf{ind}}$ is increasing.

$$\Delta\mathsf{DoF}'_{\mathsf{ind}} = \sum_{i=\lfloor \frac{N}{M} \rfloor + 1}^{K} \frac{\mathsf{B}_K(i)}{p(1-p)} (i - pK)(iM - N) \ge 0,$$

where the inequality holds since if $0 < p < \frac{N}{KM}$ and $i \ge \lfloor \frac{N}{M} \rfloor + 1$, then $i - pK \ge 0$ and $iM - N \ge 0$. For $\frac{N}{KM} \le p < 1$, similarly we can verify that $\Delta\mathsf{DoF}_{\mathsf{ind}}$ is decreasing. Both $\Delta\mathsf{DoF}_{\mathsf{dep}}$ and $\Delta\mathsf{DoF}_{\mathsf{ind}}$ increase for $0 < p < p^*$ and decrease for $p^* \le p < 1$, thus maximized at $p^* = \frac{N}{KM}$.

With $M = N = 1$ assumed, from Corollary 1, we get

$$\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{dep}} = 1 - \frac{1}{K}, \ \Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{ind}} = \left(1 - \frac{1}{K}\right)^K.$$

We can readily verify that $\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{dep}}$ grows as $K$ increases. We can also verify that $\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{ind}}$ grows as $K$ increases by showing that the ratio of $\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{ind}}$ with $K+1$ users to that with $K$ users is greater than one:

$$\begin{aligned}\frac{(1 - \frac{1}{K+1})^{K+1}}{(1 - \frac{1}{K})^K} &= \left(1 - \frac{1}{K}\right)\left(1 + \frac{1}{(K+1)(K-1)}\right)^{K+1} \\ &> \left(1 - \frac{1}{K}\right)\left(1 + \frac{1}{K-1}\right) = 1,\end{aligned}$$

where the inequality holds due to Bernoulli's inequality. Thus, both $\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{dep}}$ and $\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{ind}}$ grow as $K$ increases.

Similarly, by applying Bernoulli's inequality to the forward difference of sequence $\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{dep}} - \Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{ind}}$, we can verify that $\Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{dep}} - \Delta\mathsf{DoF}^{\mathsf{peak}}_{\mathsf{ind}}$ grows as $K$ increases.

## VI. Conclusion

We investigated the role of a relay in correlated bursty MACs where dependencies across the users' intermittent data availabilities lead to correlated transmissions across the users. We showed that the relay in most cases can provide greater DoF gains with the dependencies. Also, we demonstrated that the gap between the gain with correlated transmissions and that with uncorrelated ones can grow with more users. We found, however, that in some rare cases where the relay has very few antennas, severe collisions from the dependencies can make the relay offer less DoF gains. Considering all findings, we conclude that in general the relay's assistance is more beneficial with correlated transmissions across the users.

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