Two-way Interference Channels

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***Abstract*—We consider two-way interference channels (ICs) where forward and backward channels are ICs but not necessarily the same. We first consider a scenario where there are only two forward messages and feedback is offered through the backward IC for aiding forward-message transmission. For a linear deterministic model of this channel, we develop inner and outer bounds that match for a wide range of channel parameters. We find that the backward IC can be more efficiently used for feedback rather than if it were used for independent backward-message transmission. As a consequence, we show that feedback can provide a *net* increase in capacity even if feedback cost is taken into consideration. Moreover we extend this to a more general scenario with two additional independent backward messages, from which we find that interaction can provide an arbitrarily large gain in capacity.**

I. Introduction

The inherent two-way nature of communication links allows nodes to adapt their transmitted signals to the past received signals in exchanging their messages. Understanding the role of interaction lies at the heart of two-way communication. Since interaction is enabled through the use of feedback, *feedback* is a more basic research topic that needs to be explored towards understanding two-way communication. The history of feedback traces back to Shannon [1] who showed that feedback provides no gain in capacity for discrete memoryless point-to-point channels. Although feedback can indeed increase the capacity of multiple access channels, the increase in capacity for the Gaussian case is bounded by at most 1 bit for all channel parameters [2]. Due to these results, traditionally it is believed that interaction has had little impact on increasing capacity.

In contrast, recent research shows that feedback provides more significant gain for communication over interference channels (ICs) [3], [4]. Interestingly an explicit analysis in [4] shows that the feedback gain is unbounded, i.e., the gap between the feedback and nonfeedback capacities can be arbitrarily large for certain channel parameters. This result challenges system implementation as it assumes an idealistic feedback scenario. So a natural question that arises is to ask whether feedback provides a *net* increase in capacity even if feedback cost is taken into consideration. The first attempt to address this question has been made in [5] where it was shown that one bit of feedback is worth at most one bit of capacity, when feedback links are modeled as rate-constrained bit pipes. This implies that there is no net feedback gain unless the backward link is cheaper than the forward link.

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However, this conclusion is not guaranteed to be correct in general, since the bit-pipe modeling of the backward IC is a *separation* approach which can be suboptimal. In order to arrive at a precise conclusion, we explore two-way ICs where the backward channel is also an interference channel, but not necessarily the same as the forward channel. We first consider a simple scenario where there are only two forward messages and feedback is provided through the backward IC for helping forward-message transmission. To capture possibly different symbol rates between forward and backward channels, we introduce a parameter λ which indicates the ratio of the backward symbol rate to the forward symbol rate.

For the Avestimehr-Diggavi-Tse (ADT) deterministic model [6], we develop inner and outer bounds that match for a wide range of channel parameters. As a consequence, we find that the capacity gain due to feedback transmission can be strictly larger than the capacity gain due to another natural use of the backward channel - independent backward-message transmission. In other words, the backward IC can be more efficiently used for sending feedback signals, rather than if it were used for backward transmission of data. Hence, the overall net gain due to feedback can be still positive even if we subtract the capacity gain due to the independent backward-message transmission as a means of taking feedback cost into consideration. This shows that feedback can provide a net capacity increase. The gain comes from the fact that the backward IC's use for feedback enables the exploitation of side information at forward-message-senders to make the backward IC effectively more capable. We also extend this idea to a more general scenario where there are two additional independent backward message [7]. As a result, we show that interaction can provide an arbitrarily large gain in capacity. Moreover we find that this gain can be larger when allowing the mixture of forward-and-backward messages for transmission.

Related Work: Sahai *et.al.* [8] showed that for a two-way IC with two forward messages, there is no net feedback gain when forward-and-backward channels are the same and lie in the strong interference regime. On the other hand, we consider arbitrary forward and backward channels, and find that feedback can provide a net capacity gain for some channel regimes.

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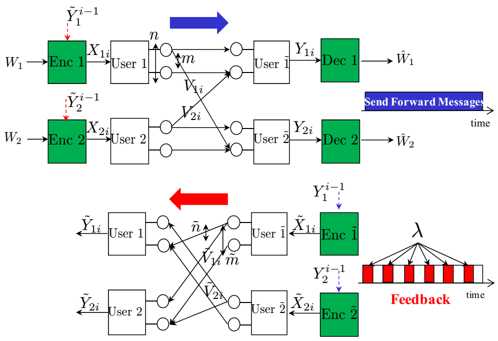

Fig. 1. Two-way ADT deterministic interference channel (IC).

II. Model

Fig. 1 describes a two-way ADT deterministic IC where user k wants to send its own message W_k to user $\tilde{k}$, and user $\tilde{k}$ feeds back a function of its received signal over the backward IC, $k = 1, 2$. We assume that W_1 and W_2 are independent and uniformly distributed. For simplicity, we consider a setting where both forward and backward ICs are symmetric but not necessarily the same. In the forward IC, n and m indicate the number of signal bit levels for direct and cross links respectively. The corresponding values in the backward IC are denoted by $(\tilde{n}, \tilde{m})$. Let $X_k \in \mathbb{F}_2^{\max(n,m)}$ be user k's transmitted signal and $V_k \in \mathbb{F}_2^m$ be a part of X_k visible to user $\tilde{j} (\neq \tilde{k})$. Similarly let $\tilde{X}_k$ be user $\tilde{k}$'s transmitted signal and $\tilde{V}_k$ be a part of $\tilde{X}_k$ visible to user $j (\neq k)$.

The encoded signal X_{ki} of user k at time i is a function of its own message and past feedback signals: $X_{ki} = f_{ki}(W_k, \tilde{Y}_k^{i-1})$. We define $\tilde{Y}_k^{i-1} := \{\tilde{Y}_{kt}\}_{t=1}^{i-1}$ where $\tilde{Y}_{kt}$ denotes the feedback signal received at user k at time t. User $\tilde{k}$'s transmitted signal $\tilde{X}_{ki}$ is a function of its past output sequences: $\tilde{X}_{ki} = \tilde{f}_{ki}(Y_k^{i-1})$. We assume possibly different symbol rates between feedback and forward channels. This difference is captured by a parameter λ which indicates the ratio of the feedback rate to the forward rate. This induces $\sum_{i=1}^N H(\tilde{X}_{ki}) \leq N\lambda \max(\tilde{n}, \tilde{m})$, where N indicates code length. For $0 \leq \lambda \leq 1$, $\tilde{X}_k^N$ is considered as a whole vector that includes feedback signals as well as null signals, e.g., $\tilde{X}_k^N = \{\varnothing, \tilde{X}_{k2}, \varnothing, \tilde{X}_{k4}, \cdots\}$. For $\lambda \geq 1$, $\tilde{X}_k^N$ contains no null signals, but each component $\tilde{X}_{ki}$ should be a larger-dimensional random vector to meet the λ-factored feedback rate. A rate pair (R_1, R_2) is said to be achievable if there exists a family of codebooks and encoder/decoder functions such that the average decoding error probabilities go to zero as code length N tends to infinity. The capacity region $\mathcal{C}$ is the closure of the set of achievable rate pairs. The sum capacity is defined as $C_{\mathsf{sum}} = \sup\{R_1 + R_2 : (R_1, R_2) \in \mathcal{C}\}$.

III. Main Results

Theorem 1 (Achievability): Let $\alpha = \frac{m}{n}$.

$$R_{\mathsf{sum}} = \begin{cases} \min\{C_{\mathsf{no}} + 2\lambda\tilde{n}, C_{\mathsf{pf}}\}, & \alpha \geq 2; \\ \min\{C_{\mathsf{no}} + 2\lambda\max(\tilde{n} - \tilde{m}, \tilde{m}), C_{\mathsf{pf}}\}, & \alpha < \frac{2}{3}, \\ C_{\mathsf{no}}, & \text{o.w.} \end{cases}$$

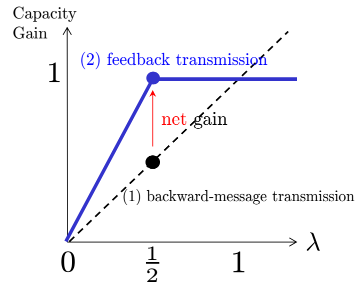

Fig. 2. Net feedback gain: $(n, m) = (2, 1)$ and $(\tilde{n}, \tilde{m}) = (1, 1)$.

where C_{no} and C_{pf} indicate the nonfeedback and perfect-feedback sum capacities respectively [9], [4]: $C_{\mathsf{no}} = \min\{2n, 2\max(n - m, m), \max(2n - m, m)\}$ and $C_{\mathsf{pf}} = \max(2n - m, m)$.

Proof: See Section IV. ∎

Theorem 2 (Outer Bound):

$$C_{\mathsf{sum}} \leq \begin{cases} \min\{C_{\mathsf{no}} + 2\lambda\tilde{n}, C_{\mathsf{pf}}\}, & \alpha \geq 2; \\ \min\{C_{\mathsf{no}} + 2\lambda\max(\tilde{n}, \tilde{m}), C_{\mathsf{pf}}\}, & \alpha < \frac{2}{3}, \\ C_{\mathsf{no}}, & \text{o.w.} \end{cases}$$

Proof: See Section V. ∎

Theorem 3 (Sum Capacity): The inner bound and the outer bound (given in Theorems 1 and 2 respectively) match and establish the sum capacity, except for the regime of $(\alpha < \frac{2}{3}, \tilde{\alpha} < 1)$, where $\tilde{\alpha} := \frac{\tilde{m}}{\tilde{n}}$.

Proof: The proof is immediate. Note that the inner and outer bounds can differ only when $\alpha < \frac{2}{3}$: the inner bound contains $2\lambda\max(\tilde{n} - \tilde{m}, \tilde{m})$, while the outer bound contains $2\lambda\max(\tilde{n}, \tilde{m})$. These two terms coincide if $\tilde{\alpha} \geq 1$; differ otherwise. ∎

Net Feedback Gain: Note that in two-way communication, there are two ways of using the backward IC: (1) Sending independent backward messages; (2) Sending feedback signals to help forward-message transmission. Using the above theorems, we will now explain why the backward IC can be more efficiently used for the second purpose, rather than if it were used for the first purpose. Consider an example where $(n, m) = (2, 1)$ and $(\tilde{n}, \tilde{m}) = (1, 1)$. Suppose that the backward channel is utilized for backward-message transmission. The capacity gain offered by the backward IC is then: $\Delta C_{\mathsf{sum}} = \lambda C_{\mathsf{B}} = \lambda$, where C_{B} denotes the nonfeedback sum capacity of the backward IC. In this example, $C_{\mathsf{B}} = 1$. Suppose that the backward channel is now used for feedback transmission. Then, due to Theorems 1 and 3, the capacity gain offered by the backward IC is: $\Delta C_{\mathsf{sum}} = \min\{2\lambda, 1\}$.

Fig. 2 plots these two capacity gains as a function of λ. Notice that when $\lambda = 0.5$, the capacity gain due to backward transmission of data is 0.5 bits; on the other hand, the capacity gain due to feedback transmission is 1 bit. Without feedback cost, the capacity gain due to feedback is 1 bit. Taking the feedback cost into consideration, we now subtract the capacity gain due to backward data transmission; hence, a net gain in capacity is $1 - 0.5 = 0.5$ bits. This implies a net feedback gain.

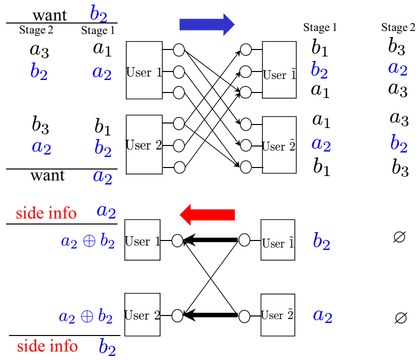

Fig. 3. Type I achievable scheme for $\alpha := \frac{m}{n} = 3$, $\tilde{\alpha} := \frac{\tilde{m}}{\tilde{n}} = 1$ and $\lambda = \frac{1}{2}$. When *side information* is exploited at user 1 and 2, the backward IC can be cast as two non-interfering point-to-point channels.

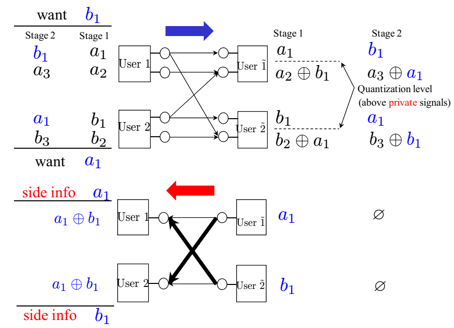

Fig. 4. Type II achievable scheme for $\alpha := \frac{m}{n} = \frac{1}{2}$, $\tilde{\alpha} := \frac{\tilde{m}}{\tilde{n}} = 1$ and $\lambda = \frac{1}{2}$.

IV. Proof of Achievability

Our feedback strategy is categorized into three types depending on the values of $(\alpha, \tilde{\alpha})$.

A. Type I: $\alpha \geq 2$

Let us explain the scheme with an example in Fig. 3, where $(n, m) = (1, 3)$, $(\tilde{n}, \tilde{m}) = (1, 1)$ and $\lambda = \frac{1}{2}$. Similar to the perfect feedback scheme, it has two stages. In the first stage, each user starts with sending $C_{\mathsf{no}}/2 = n$ bits on the upper levels. In this example, each user sends 1 bit. On the next lower level, it sends an additional bit; as a result, user 1 and 2 send (a_1, a_2) and (b_1, b_2) respectively. User $\tilde{1}$ then gets a_1 while receiving (b_1, b_2) from user 2. Similarly user $\tilde{2}$ gets b_1 and (a_1, a_2).

Similar to the perfect feedback scheme, user $\tilde{k}$ feeds back the other user's information (not received yet at the desired place). But the difference here is that this transmission is through the backward IC. Suppose that user $\tilde{1}$ and $\tilde{2}$ simultaneously send b_2 and a_2 respectively. Then, it seems impossible to decode these two bits, since each user receives the same signal. It seems that two time slots are needed to feed back these two bits. However, we can actually accomplish this in one shot. The idea is to exploit side information. Exploiting a_2 as side information, user 1 can decode b_2, and similarly user 2 can decode a_2. Here the key observation is that with *side information* at user 1 and 2, the backward IC can be viewed as *two non-interfering point-to-point channels*. In the second stage, each user sends its own fresh information on the first level and forwards the other user's information on the second level: user 1 and 2 send (a_3, b_2) and (b_3, a_2) respectively. User $\tilde{1}$ can then decode its own fresh information a_3 as well as a_2. Similarly user $\tilde{2}$ can decode (b_3, b_2). Therefore, we can achieve the sum rate of 3. Note that the backward channel is utilized once every two slots to meet the constraint of $\lambda = \frac{1}{2}$.

We now extend this to arbitrary values of (n, m), $(\tilde{n}, \tilde{m})$ and λ. In the first stage, each user starts with sending $C_{\mathsf{no}}/2 = n$ bits on the upper levels. On the next lower levels, it sends the following number of additional bits: $\min\{2\lambda\tilde{n}, C_{\mathsf{fb}} - C_{\mathsf{no}}\}$, where $C_{\mathsf{fb}} - C_{\mathsf{no}} = m - 2n$ in the regime $\alpha \geq 2$. Recall from the above example that the backward IC can be cast into two non-interfering point-to-point channels. So the effective capacity of the backward IC per user for the purpose of feedback is $\tilde{n}$. We multiply this by 2, as two stages are employed.[1] Through the backward IC, user $\tilde{k}$ can now relay the amount $\min\{2\lambda\tilde{n}, C_{\mathsf{fb}} - C_{\mathsf{no}}\}$ of the other user's information. In the second stage, each user sends $C_{\mathsf{no}}/2$ fresh bits on the upper levels and the other user information (decoded with feedback) on the next lower levels. User $\tilde{k}$ can then decode $C_{\mathsf{no}} + \min\{2\lambda\tilde{n}, C_{\mathsf{fb}} - C_{\mathsf{no}}\}$ during the two stages. Therefore, we can achieve R_{sum} in Theorem 1.

Remark 1 (Exploiting Side Information): Note in Fig. 3 that the two bits (b_2, a_2) can be fed back through the one-bit-capacity backward IC. This is because each user can cancel the seemingly interfering information by exploiting its own information as side information. This enables the net feedback gain: a capacity increase of 1 bit with $\lambda C_{\mathsf{B}} = \frac{1}{2}$ bits of the backward IC's original capability. The nature of the feedback gain offered by side information coincides with that of the two-way relay channel [10] and many other examples. □

B. Type II: $\alpha < \frac{2}{3}, \tilde{\alpha} \geq \frac{1}{2}$

We explain the second-type scheme using an example illustrated in Fig. 4. Here $(n, m) = (2, 1)$, $(\tilde{n}, \tilde{m}) = (1, 1)$ and $\lambda = \frac{1}{2}$. Similar to Type I, it has two stages. In the first stage, each user starts with sending $C_{\mathsf{no}}/2 = \max(n - m, m)$ bits, comprised of $(n - m)$ private bits and $(2m - n)^+$ common bits. In this example, each user sends one private bit only. On the upper common levels, each user sends the following number of additional bits: $\min\{2\lambda\tilde{m}, C_{\mathsf{pf}} - C_{\mathsf{no}}\}$, where $C_{\mathsf{pf}} - C_{\mathsf{no}} = \min(m, 2n - 3m)$ in the regime $\alpha < \frac{2}{3}$. In the sequel, we will show that in this regime the backward IC can be viewed as two non-interfering *cross* point-to-point channels, thus making the effective capacity of the backward IC per user $\tilde{m}$. So we have used the $2\lambda\tilde{m}$ for the number of additional bits. Similarly, a factor of 2 is multiplied due to the two-stage nature of the scheme. In this example, $2\lambda\tilde{m} = 1$. So user 1 and 2 send a_1 and b_1 respectively.

[1] Here $2\lambda\tilde{n}$ can be a non-integer rational number, which is incompatible with an input of the ADT model. However, we can resolve this by employing multiple time slots (say M) within each stage to make $2\lambda\tilde{n}M$ an integer.

In the perfect feedback scheme, user 1 wanted to know the other user information b_1. Similarly user 2 wanted to know a_1. And to satisfy this demand, the two bits $(a_2 \oplus b_1, b_2 \oplus a_1)$ were fed back to the users. Suppose we mimic this transmission: user $\tilde{1}$ and $\tilde{2}$ send $a_2 \oplus b_1$ and $b_2 \oplus a_1$ respectively. This does not work though. User 1 cannot decode b_1 from the received signal $a_1 \oplus a_2 \oplus b_1 \oplus b_2$ and similarly user 2 cannot decode a_1. It looks two time slots are needed to feed back these two bits. However, we can satisfy the demand in one shot. Note that the symbol b_1 wanted by user 1 is available at user $\tilde{2}$. Similarly the symbol a_1 wanted by user 2 is available at user $\tilde{1}$. Suppose we now send these two bits instead. User 1 and 2 can then decode b_1 and a_1 respectively, exploiting its own signal as side information. The key observation here is that exploiting side information at user 1 and 2, the backward IC becomes equivalent to two non-interfering cross point-to-point channels. This enables feeding back the following number of bits: $\min\{2\lambda\tilde{m}, C_{\sf pf} - C_{\sf no}\}$. In the second stage, each user starts with the nonfeedback scheme and additionally sends the other user's information (decoded with feedback) on vacant common levels. User $\tilde{k}$ can then decode $C_{\sf no} + \min\{2\lambda\tilde{m}, C_{\sf fb} - C_{\sf no}\}$ bits during the two stages, thus achieving $R_{\sf sum}$ in Theorem 1.

C. Type III: $\alpha < \frac{2}{3}, \tilde{\alpha} < \frac{1}{2}$

The only distinction with respect to Type II is that in this regime of $\tilde{\alpha} < \frac{1}{2}$, the effective capacity of the backward IC per user for the purpose of feedback is now $\tilde{n} - \tilde{m}$. This is because the backward IC is now equivalent to two point-to-point channels *composed of private levels only*. In Fig. 4, remember that user $\tilde{1}$ fed back a_1 to user 2 through the *cross* link, so the transmission rate was limited by $\tilde{m}$. However, in the regime of $\tilde{\alpha} < \frac{1}{2}$, $\tilde{n} - \tilde{m} > \tilde{m}$. This motivates us to consider a better alternative: user $\tilde{k}$ uses $\tilde{n} - \tilde{m}$ private levels for feedback. For example, user $\tilde{1}$ can alternatively feed back $a_2 \oplus b_1$ using a private level, thus allowing user 1 to decode b_1. Taking this alternative, the effective capacity of the backward IC per user for the purpose of feedback is $\tilde{n} - \tilde{m}$. This way, user $\tilde{k}$ can decode $C_{\sf no} + \min\{2\lambda(\tilde{n} - \tilde{m}), C_{\sf fb} - C_{\sf no}\}$ bits during the two stages. This completes the proof.

D. Translation to the Gaussian channel

The deterministic-channel achievability gives insights into an achievable scheme in the Gaussian channel. This is inspired by several observations that can be made from the example in Fig. 4. The first observation is that the fed back information a_1 from user $\tilde{1}$ can be interpreted as a quantized version of the received signal $(a_1, a_2 \oplus b_1)^t$ at the level above the corrupted *private* signal $a_2 \oplus b_1$. Second, the b_1 sent by user 1 at stage 2 can be interpreted as a block-Marov-encoded signal of $a_1 \oplus b_1$ conditioned on the previously-sent information a_1. Lastly, a_3 is new fresh information superimposed on b_1. These observations motivate us to employ quantize-map-and-forward [6], [11] for feedback, and block Markov encoding and superposition schemes at forward-message senders. Here the quantization level should be carefully chosen depending on channel parameter regimes. In this example, we quantize the received signals above the level of the corrupted private signal, since the private signal is not desirable at the other user. For other regimes, a careful choice needs be made on the quantization level accordingly, in order to well mimick the deterministic-channel achievability.

V. PROOF OF OUTER BOUND

The proof of $C_{\sf sum} \le 2n + 2\lambda\tilde{n}$ is based on the standard cutset argument. Also note that the perfect-feedback bound $C_{\sf sum} \le C_{\sf pf}$ is due to [4], [8]. For completeness, we include these proofs in the full version [12]. The main focus here is to prove $C_{\sf sum} \le 2\max(n-m, m) + 2\lambda\max(\tilde{n}, \tilde{m})$. Starting with Fano's inequality, we get

$$
\begin{aligned}
&N(R_1 + R_2 - \epsilon_N) \le I(W_1; Y_1^N) + I(W_2; Y_2^N) \\
&= H(Y_1^N) - H(Y_1^N|W_1) + H(Y_2^N) - H(Y_2^N|W_2) \\
&\overset{(a)}{=} H(Y_1^N, V_1^N) - H(V_1^N) + H(Y_2^N, V_2^N) - H(V_2^N) \\
&+ \left\{H(V_1^N) - H(Y_1^N|W_1) - H(V_1^N|Y_1^N)\right\} \\
&+ \left\{H(V_2^N) - H(Y_2^N|W_2) - H(V_2^N|Y_2^N)\right\} \\
&\overset{(b)}{=} H(Y_1^N|V_1^N) + H(Y_2^N|V_2^N) \\
&+ \left\{I(V_1^N; W_2) + H(V_2^N|W_1) - H(Y_1^N|W_1) - H(V_1^N|Y_1^N)\right\} \\
&+ \left\{I(V_2^N; W_1) + H(V_1^N|W_2) - H(Y_2^N|W_2) - H(V_2^N|Y_2^N)\right\} \\
&\overset{(c)}{\le} H(Y_1^N|V_1^N) + H(Y_2^N|V_2^N) + H(\tilde{Y}_1^N|W_1) + H(\tilde{Y}_2^N|W_2) \\
&\le \sum \left\{H(Y_{1i}|V_{1i}) + H(Y_{2i}|V_{2i}) + H(\tilde{Y}_{1i}) + H(\tilde{Y}_{2i})\right\} \\
&\overset{(d)}{\le} 2N\max(n-m, m) + 2N\lambda\max(\tilde{n}, \tilde{m}),
\end{aligned}
$$

where (a) and (b) follow from a chain rule; (c) follows from Claim 1 (see below); (d) follows from $\sum H(\tilde{Y}_{ki}) \le N\lambda\max(\tilde{n}, \tilde{m})$. If (R_1, R_2) is achievable, then $\epsilon_N \to 0$ as N tends to infinity. Therefore, we get the desired bound.

Claim 1: For $(k, \ell) = (1, 2)$ or $(k, \ell) = (2, 1)$,

$$
\begin{aligned}
&I(V_k^N; W_\ell) + H(V_\ell^N|W_k) - H(Y_k^N|W_k) - H(V_k^N|Y_k^N) \\
&\le H(\tilde{Y}_k^N|W_k).
\end{aligned}
$$

Proof: By symmetry, it is enough to prove only one case.

$$
\begin{aligned}
&I(V_1^N; W_2) + H(V_2^N|W_1) - H(Y_1^N|W_1) - H(V_1^N|Y_1^N) \\
&\overset{(a)}{=} I(V_1^N; W_2) + \left\{H(V_2^N|W_1, \tilde{Y}_1^N) - H(Y_1^N|W_1, \tilde{Y}_1^N)\right\} \\
&\quad + I(V_2^N; \tilde{Y}_1^N|W_1) - I(Y_1^N; \tilde{Y}_1^N|W_1) - H(V_1^N|Y_1^N) \\
&\overset{(b)}{=} I(V_1^N; W_2) + H(\tilde{Y}_1^N|W_1, Y_1^N) \\
&\quad - H(\tilde{Y}_1^N|W_1, V_2^N) - H(V_1^N|Y_1^N) \\
&\overset{(c)}{\le} I(V_1^N; W_2) + H(\tilde{Y}_1^N|W_1, Y_1^N, V_1^N) \\
&\le I(W_1, V_1^N; W_2) + H(\tilde{Y}_1^N|W_1, V_1^N) = H(\tilde{Y}_1^N, V_1^N|W_1) \\
&\overset{(d)}{=} H(\tilde{Y}_1^N|W_1)
\end{aligned}
$$

where (a) follows from a chain rule; (b) follows from $H(Y_1^N|W_1, \tilde{Y}_1^N) = H(V_2^N|W_1, \tilde{Y}_1^N)$ due to the fact that X_1^N is a function of $(W_1, \tilde{Y}_1^{N-1})$; (c) follows from a chain rule and the fact that entropy is non-negative; (d) follows from the fact that V_1^N is a function of $(W_1, \tilde{Y}_1^{N-1})$. ∎

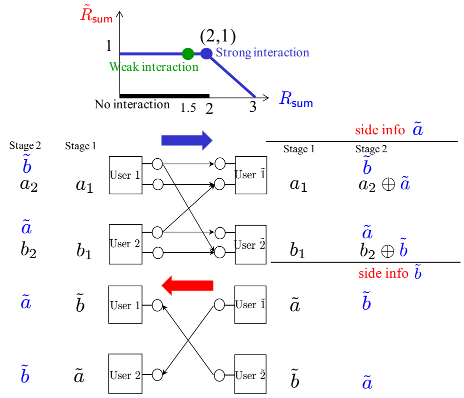

Fig. 5. Two-way interference channel with four messages and $\lambda = 1$. Illustration of an achievable scheme for $(R_{\text{sum}}, \tilde{R}_{\text{sum}}) = (2, 1)$.

VI. Two-way IC with Four Messages

We now consider a more general scenario where there are four messages in total: two forward messages; and two backward messages from user $\tilde{k}$ to user k, $k = 1, 2$. In this scenario, the encoded signal $\tilde{X}_{ki}$ of user $\tilde{k}$ is now a function of $(\tilde{W}_k, Y_{\tilde{k}}^{i-1})$. We will demonstrate from an example in Fig. 5 that interaction can improve the non-interactive rate significantly. For simplicity, we focus on a sum-rate pair of the forward and backward messages, denoted by $(R_{\text{sum}}, \tilde{R}_{\text{sum}}) := (R_1 + R_2, \tilde{R}_1 + \tilde{R}_2)$. Note that the non-interactive capacity region is $\{(R_{\text{sum}}, \tilde{R}_{\text{sum}}) : R_{\text{sum}} \leq 2, \tilde{R}_{\text{sum}} = 0\}$ [9]. On the other hand, we will show that interaction gives: $\{(R_{\text{sum}}, \tilde{R}_{\text{sum}}) : \tilde{R}_{\text{sum}} \leq 1, R_{\text{sum}} + \tilde{R}_{\text{sum}} \leq 3\}$. With Type-I and Type-II schemes in Section IV, we can achieve the $(0, 1)$ and $(3, 0)$ points respectively. On the other hand, a new idea emerges to achieve a corner point of $(2, 1)$.

The example in Fig. 5 shows an achievable scheme for the $(2, 1)$ point. Here we will demonstrate that during two stages, user 1 and 2 can send (a_1, a_2) and (b_1, b_2) respectively, while user $\tilde{1}$ and $\tilde{2}$ can transmit $\tilde{a}$ and $\tilde{b}$ respectively. In the first stage, user 1 and 2 send a_1 and b_1 using its own private level respectively. Meanwhile user $\tilde{1}$ and $\tilde{2}$ send $\tilde{a}$ and $\tilde{b}$ respectively through the backward IC. User 1 then gets the unwanted information $\tilde{b}$ and similarly user 2 receives $\tilde{a}$. In the second stage, through the forward IC, user 1 feeds $\tilde{b}$ back to user $\tilde{1}$ using the top level, and similarly user 2 feeds $\tilde{a}$ back to user $\tilde{2}$. Here the key observation is that this feedback transmission comes *for free*, i.e., it does not hurt forward-message transmission of (a_2, b_2). Notice that $\tilde{a}$ and $\tilde{b}$ are user $\tilde{1}$'s and $\tilde{2}$'s own information respectively. This allows user 1 and 2 to send their own forward information a_2 and b_2 without being interfered. In other words, exploiting $\tilde{a}$ as side information, user $\tilde{1}$ can decode a_2, and similarly user $\tilde{2}$ can decode b_2. Upon receiving $(\tilde{b}, \tilde{a})$, user $\tilde{1}$ and $\tilde{2}$ transmit the other user's information respectively.[2] This transmission enables user 1 and 2 to decode their desired signals, thus achieving $(2, 1)$.

[2]In fact, the transmission of $(\tilde{b}, \tilde{a})$ must occur in a later stage due to delay. In stage 2, we should instead transmit new fresh information and perform a pipelined procedure. It turns out this precise scheme can also achieve $(2, 1)$. For a simpler description, however, we ignored the delay.

Mixing Forward-and-Backward Messages: Interestingly, this interactive scheme includes the mixture of forward-and-backward messages. Note that in the second stage, user 1 sends $(\tilde{b}, a_2)$ at the same time. We say that interaction is *strong* if the mixture is allowed. On the other hand, if the mixture is not allowed (that we call *weak* interaction), the performance is degraded. For example, we can show that given the constraint of $\tilde{R}_{\text{sum}} = 1$, the weak interaction provides at most 1.5 bits for R_{sum}. See [12] for the proof. Therefore, interaction can provide a larger gain when allowing for the mixture of different messages.

Remark 2 (System Implication): Feedback gain occurs when forward and backward ICs are different. This occurs naturally in FDD systems. This can also be created by opportunistically pairing of subbands for the forward and backward transmissions. □

VII. Conclusion

For the two-way ADT deterministic IC where feedback is offered through the backward IC, we developed three types of achievable schemes and derived outer bounds, thereby establishing the sum capacity except for the regime of $(\alpha < \frac{2}{3}, \tilde{\alpha} < 1)$. As a consequence, we showed that there is net feedback gain for some channel regimes. Our future work is: (1) Translating to the Gaussian channel with the guideline in Section IV-D; (2) Extending to general channel settings.

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