

# A Relay Can Increase Degrees of Freedom in Bursty MIMO Interference Networks

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**Abstract**—We explore the benefits of relays<sup>1</sup> in multi-user wireless networks with bursty user traffic, where intermittent data traffic restricts the users to bursty transmissions. Specifically, we investigate a two-user *bursty* MIMO Gaussian interference channel (IC) with a relay, where two Bernoulli random states govern the bursty user traffic. We show that an in-band relay can provide a degrees of freedom (DoF) gain in this bursty channel. This beneficial role is in contrast to the role in the non-bursty channel which is not as significant to provide a DoF gain. More importantly, we demonstrate that for certain antenna configurations, an in-band relay can help achieve an *interference-free* performance with increased DoF. Particularly, we find the benefits of a relay substantial with low user traffic, as the DoF gain can scale *linearly* with the number of antennas at the relay. To this end, we derive an outer bound from which we obtain a necessary condition for interference-free DoF performances. Then, we develop a novel scheme that exploits information of the bursty traffic states to achieve the performances.

## I. INTRODUCTION

As availability of wireless systems prevails along with an increasing number of mobile devices, interference is inevitably everywhere. Incorporating a relay in wireless systems has been widely considered as a promising means to mitigate such interference. Although it has been revealed that adding relays can indeed help mitigate interference, the performance gain turns out to be insignificant. In particular, it has been proved that adding in-band relays does not increase the number of degrees of freedom (DoF) in interference networks [5].

But it is premature to conclude that relays play little role in interference networks. The result is based on the conventional assumption in finding the fundamental limits of interference networks, which is the constant presence of interference. A rationale behind it is that a sufficient amount of data is always available, thus transmissions occur all the time. However, in practical systems, intermittent data traffic restricts the amount of data, thus bursty transmissions take place. It is in fact such *burstiness* that needs to be considered to investigate the practical benefit of employing a relay in interference networks.

From an observation in a simple single-user setting, one can expect a significant benefit of adding relays in bursty traffic networks. To see this, consider a *bursty* MIMO relay channel (RC), where the transmitter sends signals with probability  $p$ ;

<sup>1</sup>In this work, we focus on the role of *in-band* relays although out-of-band relays have also been of interest [1]–[4].

the transmitter has a large number of antennas, and the receiver and the relay have 1 and  $L$  respectively. The standard cut-set argument yields a bound on the DoF:  $\min\{p(1+L), 1\}$ , which is also achievable as the cut-set bound is well known to be tight in single-source single-destination networks. Observe that the DoF is  $p$  without a relay ( $L = 0$ ), and it is strictly greater with a relay ( $L \geq 1$ ). From this observation, we see that a relay in this bursty single-user channel can provide a DoF gain, and also the gain can scale linearly with  $L$  especially in low-traffic regimes where  $p \ll 1$ . This promising result in the single-user setting motivates us to explore further the role of relays in multi-user bursty traffic networks. Specifically we ask: can relays play a significant role in *bursty* interference networks to achieve an *interference-free* DoF performance?

To answer this question, we consider a two-user *bursty* MIMO Gaussian interference channel (IC) with a relay, where two independent Bernoulli random processes govern the bursty data traffic of the users. We derive an outer bound, from which we obtain a necessary condition on the antenna configuration for interference-free performances. Also, we develop a novel scheme that harnesses information of the bursty traffic states. Through this information, our scheme enables the relay and the transmitters to cooperate in a beneficial manner, thus providing a significant DoF gain over the channel without a relay. More importantly, our scheme reveals that a relay can help achieve an *interference-free* DoF performance. This result discovers that the role of relays in the bursty channel is crucial in contrast to that in the non-bursty channel, which is not as considerable to increase DoF. We find the presence of a relay particularly beneficial with low user traffic, as the DoF gain can scale *linearly* with the number of antennas at the relay.

**Related Work:** Substantial work has been done toward understanding the Gaussian IC with a relay. Numerous techniques developed in the IC and the RC have been applied to this channel, considering various types of relays [1]–[7]. One distinction of our work to the past works is that we look into more realistic traffic scenarios where *bursty* transmissions can take place. Considering such scenarios, Wang and Diggavi first investigated a bursty Gaussian IC without a relay [8]. Extending the result in [8] to the MIMO channel with a relay, we demonstrate that the benefits of relays in the bursty MIMO Gaussian IC are significant.

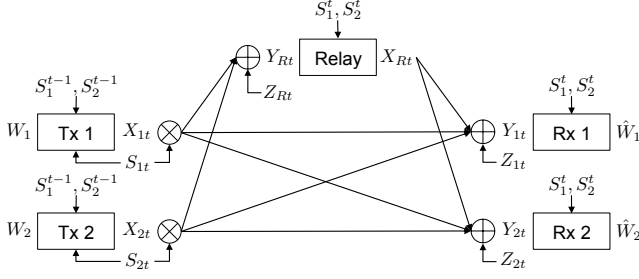


Fig. 1. Bursty MIMO Gaussian interference channel with a relay

## II. MODEL

Fig. 1 describes the bursty MIMO Gaussian interference channel with a relay. Transmitter  $k$  wishes to deliver message  $W_k$  reliably to receiver  $k$ ,  $\forall k = 1, 2$ . Let  $X_{kt} \in \mathbb{C}^M$  be the encoded signal of transmitter  $k$  at time  $t$ , and  $X_{Rt} \in \mathbb{C}^L$  be the encoded signal of the relay at time  $t$ . Traffic states  $S_{kt}$  are assumed to be independent,  $\text{Bern}(p)$ , and i.i.d. over time to capture uncoordinated bursty transmissions<sup>2</sup>. But the relay is not limited to bursty transmissions. It aims to help transmitter-receiver links based on its past received signals, thus it can send signals at all times as long as it has past received signals. Additive noise terms  $Z_{kt}$  and  $Z_{Rt}$  are assumed to be independent,  $\mathcal{CN}(0, \mathbf{I}_N)$  and  $\mathcal{CN}(0, \mathbf{I}_L)$ , and i.i.d. over time. Let  $Y_{kt} \in \mathbb{C}^N$  be the received signal of receiver  $k$  at time  $t$ , and  $Y_{Rt} \in \mathbb{C}^L$  be the received signal of the relay at time  $t$ .

$$Y_{kt} = \mathbf{H}_{k1}S_{1t}X_{1t} + \mathbf{H}_{k2}S_{2t}X_{2t} + \mathbf{H}_{kR}X_{Rt} + Z_{kt},$$

$$Y_{Rt} = \mathbf{H}_{R1}S_{1t}X_{1t} + \mathbf{H}_{R2}S_{2t}X_{2t} + Z_{Rt}.$$

The matrices  $\mathbf{H}_{ji}$ ,  $\mathbf{H}_{Ri}$ , and  $\mathbf{H}_{jR}$  describe the time-invariant channels from transmitter  $i$  to receiver  $j$ , from transmitter  $i$  to the relay, and from the relay to receiver  $j$ ,  $\forall i, j = 1, 2$ . All channel matrices are assumed to be full rank. We assume current traffic states are available at the receivers and the relay, as receiving ends can detect which transmitting end is active, for instance, by measuring the energy levels of incoming signals. We also assume the transmitters get feedback of past traffic states from the receivers. Each transmitter knows its current traffic state as it processes the arrivals of data for transmission. Transmitter  $k$  generates its encoded signal at time  $t$  based on its own message, its current traffic state, and the feedback of past traffic states:  $X_{kt} = f_{kt}(W_k, S_{kt}, S^{t-1})$ , where  $S_t$  stands for  $(S_{1t}, S_{2t})$  and  $S^{t-1}$  stands for the sequence up to  $t-1$ . The relay generates its encoded signal at time  $t$  based on its past received signals, and both past and current traffic states:  $X_{Rt} = f_{Rt}(Y_{Rt}^{t-1}, S^t)$ . We define the DoF region  $\mathcal{D} = \{(d_1, d_2) : \exists (R_1, R_2) \in \mathcal{C}(P) \text{ such that } d_k = \lim_{P \rightarrow \infty} \frac{R_k}{\log P}\}$ , where  $\mathcal{C}(P)$  is the capacity region with power constraint  $\bar{P}$ .

<sup>2</sup>Finite-size buffers at nodes can be another source of burstiness, as they limit the amount of data available for transmission and reception. In this work, we consider intermittent data traffic to be a primary source of burstiness. Also, we assume uncoordinated transmissions, as distributed media access control protocols can lead to imperfect coordination between multiple nodes.

## III. MAIN RESULTS

For completeness, we first describe the following result for the single-user case, which is immediate since the cut-set bound is tight in terms of DoF in single-user networks [9].

*Theorem 1:* The DoF of the bursty MIMO Gaussian relay channel is characterized by

$$d = \min \left\{ p \min(M, N + L), p \min(M + L, N) + (1 - p) \min(L, N) \right\}.$$

Next, we present our main results for the bursty MIMO Gaussian interference channel with a relay.

*Theorem 2:* A DoF outer bound of the bursty MIMO Gaussian IC with a relay is

$$d_1, d_2 \leq \min \left\{ \begin{array}{l} p \min(M, N + L), \\ p \min(M + L, N) \\ + (1 - p) \min(L, N) \end{array} \right\}, \quad (1)$$

$$d_1 + d_2 \leq \min \left\{ \begin{array}{l} p \min\{(M - N)^+, N + L\}, \\ p \min\{(M + L - N)^+, N\} \\ + (1 - p) \min\{(L - N)^+, N\} \end{array} \right\}$$

$$+ \left\{ \begin{array}{l} p^2 \min(2M + L, N) \\ + 2p(1 - p) \min(M + L, N) \\ + (1 - p)^2 \min(L, N) \end{array} \right\}. \quad (2)$$

*Proof:* See Section V. ■

The above bound recovers the DoF results in the non-bursty case ( $p = 1$ ) [5] and the case without a relay ( $L = 0$ ) [8].

Using this bound, we obtain a necessary condition for attaining interference-free DoF. This is done by examining when (2) becomes inactive. The proof can be found in [10].

*Corollary 1:* A necessary condition for attaining interference-free DoF is the union of three conditions  $\mathcal{C}_1$ ,  $\mathcal{C}_2$ , and  $\mathcal{C}_3$  below:

$$\mathcal{C}_1 : 2M \leq N,$$

$$\mathcal{C}_2 : M \geq 2N + L \text{ and } L \geq 2N,$$

$$\mathcal{C}_3 : M \geq 2N \text{ and } 3L \leq N.$$

Finally, we establish a sufficient condition for attaining interference-free DoF.

*Theorem 3:* A sufficient condition for attaining interference-free DoF is the union of three conditions  $\mathcal{C}_1, \mathcal{C}_2$  above, and  $\mathcal{C}'_3$  below:

$$\mathcal{C}'_3 : M \geq 2N + L \text{ and } 3L \leq N.$$

*Proof:* See Section IV. ■

## IV. PROOF OF THEOREM 3

In this section, we develop an explicit scheme that achieves interference-free DoF in the bursty MIMO Gaussian IC with a relay. We consider two different regimes in this section depending on the level of data traffic.

- **Low-traffic regime:** Low data traffic limits the information flow at the transmitters. Thus the transmitters send as much information as possible per active transmission toward the intended receivers.

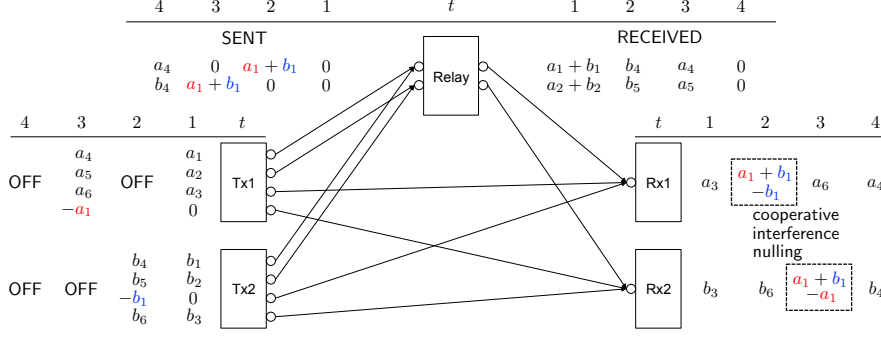


Fig. 2. An achievable scheme for  $(M, N, L) = (4, 1, 2)$  configuration

- **High-traffic regime:** High data traffic may limit the information flow at the receivers, especially when they have a small number of antennas. In this case, the transmitters reduce the amount of information sent per active transmission to ensure the decoding at the receivers.

#### A. $2M \leq N$

Both transmitters always send  $M$  fresh symbols, and the relay sends nothing. Since each receiver has a sufficient number of antennas, it decodes desired symbols whenever its corresponding transmitter is active. This scheme achieves the following DoF region:  $\mathcal{D} = \{(d_1, d_2) : d_1, d_2 \leq pM\}$ .

#### B. $M \geq 2N + L$ and $L \geq 2N$

We present the scheme with an example for the simplest configuration:  $(M, N, L) = (4, 1, 2)$ . The generalization of the scheme is in [10]. Fig. 2 demonstrates how the transmitters and the relay operate, with an example sequence of traffic states  $(S_1, S_2)$ :  $(1, 1), (0, 1), (1, 0), (0, 0)$ . The transmitters and the relay always apply zero-forcing precoding. The connected links in Fig. 2 depict the effect of the precoding.

**Low-traffic regime**  $p < \frac{1}{3}$ : Each transmitter sends one fresh symbol to its corresponding receiver and two to the relay at the rate of  $p$ . The use of one extra antenna is to send symbols to the other receiver when *cooperation* with the relay is needed.

- **Time 1:** Both transmitters send three fresh symbols. Knowing both are active, the relay sends nothing. Two unavoidable collisions take place at the relay:  $a_1 + b_1$  and  $a_2 + b_2$ .
- **Time 2:** From feedback, the transmitters are aware of the past collisions. To remove its footprint in the collisions, transmitter 2 sends  $-b_1$  to receiver 1, in addition to three fresh symbols toward receiver 2 and the relay. Knowing that, the relay sends  $a_1 + b_1$  to receiver 1 only. Receiver 1 gets  $a_1$ , the sum of  $-b_1$  and  $a_1 + b_1$ . The relay and transmitter 2 *cooperate* and deliver only desired information to receiver 1 with interference removed.
- **Time 3:** Knowing transmitter 1 is active, the relay cooperates with transmitter 1. This delivers  $b_1$  to receiver 2.
- **Time 4:** Both transmitters are inactive, and the relay has no active transmitter to cooperate with. The relay delivers the past reserved symbols that were not collided. It sends  $a_4$  to receiver 1 and  $b_4$  to receiver 2.

The proposed scheme works when relay-passing symbols are delivered to the receivers faster than they build up at the relay. Let us perform an analysis with two types of user 1's relay-passing symbols.

- **Type 1 symbols:** collision-free ( $a_4$  and  $a_5$ ). Two symbols are reserved with probability  $p(1-p)$  (only transmitter 1 is active), and one can be delivered to receiver 1 with probability  $(1-p)^2$  (both transmitters are inactive). Type 1 symbols do not build up at the relay if  $p(1-p) \times 2 < (1-p)^2 \times 1$ .
- **Type 2 symbols:** collisions ( $a_1$  and  $a_2$ ). Two symbols are reserved with probability  $p^2$  (both transmitters are active), and one can be delivered to receiver 1 with probability  $(1-p)p$  (only transmitter 2 is active). Type 2 symbols do not build up at the relay if  $p^2 \times 2 < (1-p)p \times 1$ .

In the low-traffic regime where  $p < \frac{1}{3}$ , the above conditions hold. Each transmitter sends 3 fresh symbols at the rate of  $p$ , and all of them will be eventually decoded at the corresponding receiver: the DoF of  $3p$ .

**High-traffic regime**  $\frac{1}{3} \leq p < 1$ : Both transmitters send fresh symbols at a lower rate; they choose to send symbols with probability  $q$  at any time instant. Therefore, each transmitter is in fact active with probability  $pq$ . A similar analysis by replacing  $p$  with  $pq$  gives the following conditions.

- Type 1 symbols:  $(pq)(1-pq) \times 2 < (1-pq)^2 \times 1$ .
- Type 2 symbols:  $(pq)^2 \times 2 < (1-pq)(pq) \times 1$ .

In the high-traffic regime where  $\frac{1}{3} \leq p < 1$ , defining  $q$  as  $\frac{1}{p}(\frac{1}{3} - \epsilon)$ , where  $\epsilon > 0$ , satisfies the above conditions. Each transmitter sends 3 fresh symbols at the rate of  $pq$ , and all of them will be eventually decoded at the corresponding receiver. As both transmitters choose  $\epsilon$  arbitrarily close to zero, the DoF converges to 1. The proposed scheme achieves the DoF region  $\mathcal{D} = \{(d_1, d_2) : d_1, d_2 \leq \min(3p, 1)\}$ .

**Remarks (Cooperative Interference Nulling):** From user 1's perspective, to achieve interference-free DoF, the transmitter always send three fresh symbols: one directly to the intended receiver and the other two to the relay. The relay later delivers them when the transmitter is inactive. Since the relay is shared, these relay-passing symbols sometimes get interfered. But they are delivered to the intended receiver *interference-free*. At Time 2, for example, when transmitter 1 is inactive, the relay and transmitter 2 cooperate and remove

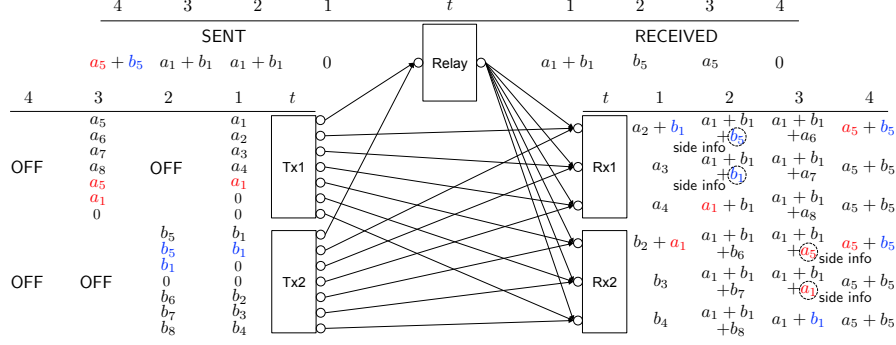


Fig. 3. An achievable scheme for  $(M, N, L) = (7, 3, 1)$  configuration

interference to deliver  $a_1$  to receiver 1. When both transmitters are inactive, the relay applies zero-forcing precoding and delivers relay-passing symbols that were not interfered, for example,  $a_4$  at Time 4. Overall, all symbols of user 1 are delivered to the intended receiver without interference at all times. Cooperative interference nulling is a notable distinction from other schemes, because the relay and the transmitters *synchronously cooperate* by exploiting current traffic states.

### C. $M \geq 2N + L$ and $3L \leq N$

We present the scheme with an example for the simplest configuration:  $(M, N, L) = (7, 3, 1)$ . The generalization of the scheme is in [10]. Fig. 3 demonstrates how the transmitters and the relay operate, with an example sequence of traffic states  $(S_1, S_2)$ :  $(1, 1), (0, 1), (1, 0), (0, 0)$ . The transmitters always apply zero-forcing precoding, whereas the relay broadcasts. The connected links in Fig. 3 depict the effect of the precoding.

**Low-traffic regime**  $p < \frac{1}{2}$ : Each transmitter sends three fresh symbols to its corresponding receiver and one to the relay at the rate of  $p$ . The use of three extra antennas is to provide *side information* to the other receiver.

- **Time 1:** Both transmitters send four fresh symbols. In addition, each transmitter sends to the other receiver the duplicate of its relay-passing symbol to provide side information, which will be used to resolve interference. There is one collision  $a_1 + b_1$  at the relay, and one at each receiver:  $a_2 + b_1$  and  $b_2 + a_1$ . Each receiver needs to resolve its collision to decode its desired symbol:  $a_2$  and  $b_2$ .
- **Time 2:** Besides four fresh symbols, transmitter 2 sends  $b_1$  and  $b_5$ , the duplicate of its relay-passing symbols, to receiver 1 to provide side information. The relay sends  $a_1 + b_1$ , and it is broadcast to both receivers. Receiver 1 decodes  $a_1, b_1$ , and  $b_5$ . Receiver 1 resolves past collision  $a_2 + b_1$  using side information  $b_1$ .  $b_5$  will be used later.
- **Time 3:** Transmitter 1 is active. Similarly, receiver 2 decodes  $b_1, a_1$ , and  $a_5$ . Receiver 2 resolves past collision  $b_2 + a_1$  using side information  $a_1$ .  $a_5$  will be used later.
- **Time 4:** Both transmitters are inactive. The relay sends the sum of  $a_5$  and  $b_5$  to deliver information that is useful to both receivers. Receiver 1 decodes  $a_5$  from  $a_5 + b_5$  since it has  $b_5$  as side information, and receiver 2 decodes  $b_5$ .

The proposed scheme works when relay-passing symbols are delivered to the receivers faster than they build up at the relay. Also, each user needs *all* relay-passing symbols of the other user as side information, because they are broadcast by the relay, and cause interference. Let us perform an analysis from user 1's perspective.

- User 1's relay-passing symbols ( $a_1$  and  $a_5$ ): one symbol is reserved with probability  $p$  (Transmitter 1 is active), and can be delivered to receiver 1 with probability  $1 - p$  (Transmitter 1 is inactive). They do not build up at the relay if  $p \times 1 < (1 - p) \times 1$ .
- User 2's relay-passing symbols ( $b_1$  and  $b_5$ ): one symbol is reserved with probability  $p$  (Transmitter 2 is active), and eventually broadcast. This causes interference. Receiver 1 can get the duplicate of two user 2's relay-passing symbols as side information from transmitter 2 with probability  $(1 - p)p$  (only transmitter 2 is active). This can be used to resolve interference. User 1 decodes all user 2's relay-passing symbols if  $p \times 1 < (1 - p)p \times 2$ .

In the low-traffic regime where  $p < \frac{1}{2}$ , the above conditions hold. Each transmitter sends 4 fresh symbols at the rate of  $p$ , and all of them will be eventually decoded at the corresponding receiver in the low-traffic regime: the DoF of  $4p$ .

**High-traffic regime**  $\frac{1}{2} \leq p < 1$ : Both transmitters send fresh symbols to the relay at a lower rate; they choose to send symbols to the relay with probability  $q$  at any time instant. Therefore, each transmitter sends symbols to its corresponding receiver at the rate of  $p$ , and to the relay at the rate of  $pq$ . We can perform a similar analysis from user 1's perspective.

- User 1's relay-passing symbols are reserved at the rate of  $pq \times 1$ , and can be delivered at the rate of  $(1 - p) \times 1$ . They do not build up at the relay if  $pq \times 1 < (1 - p) \times 1$ .
  - User 2's relay-passing symbols are reserved at the rate of  $pq \times 1$ , and eventually broadcast. Receiver 1 can get the duplicate of user 2's relay-passing symbols as side information from transmitter 2 at the rate of  $(1 - p)p \times 2$ . User 1 decodes all user 2's relay-passing symbols if  $pq \times 1 < (1 - p)p \times 2$ .
- In the high-traffic regime where  $\frac{1}{2} \leq p < 1$ , defining  $q$  as  $\frac{1}{p}(1 - p - \epsilon)$ , where  $\epsilon > 0$ , satisfies the above conditions. Each transmitter sends 3 fresh symbols to its corresponding receiver at the rate of  $p$ , and 1 fresh symbol to the relay at the rate of  $pq$ .

All of them will be eventually decoded at the corresponding receiver. As both transmitters choose  $\epsilon$  arbitrarily close to zero, the DoF converges to  $2p + 1$ . The proposed scheme achieves the DoF region  $\mathcal{D} = \{(d_1, d_2) : d_1, d_2 \leq \min(4p, 2p + 1)\}$ .

**Remarks (Exploiting Side Information):** The relay has a limited number of antennas, thus its transmitted symbols are broadcast to both receivers. Each receiver gets undesired relay-passing symbols of the other user. To resolve the interference, each transmitter provides the other receiver with *side information*. At *Time 2*, for example, transmitter 2 provides  $b_1$  and  $b_5$  to receiver 1, and receiver 1 uses them to resolve the interference in  $a_2 + b_1$  and  $a_5 + b_5$ . Overall, each receiver gets its desired symbols, some of which are possibly interfered by relay-passing symbols of the other user. With side information provided by the other transmitter, each receiver can resolve such interference and decode its desired symbols. Gains obtained by exploiting side information appear in many other network examples [11].

## V. PROOF OF THEOREM 2

The bound (1) is the cut-set bound, so we omit the proof. The bound (2) consists of two bounds, and we derive one of them in this section. The other can be derived similarly. For the complete proof, see [10]. The outer bound proof follows the genie-aided approach. For notational convenience, let  $\sum$  denote  $\sum_{t=1}^n$ ,  $S_t$  denote  $(S_{1t}, S_{2t})$ , and  $S^n$  denote the sequence of  $S$  up to  $n$ .

$$\begin{aligned}
n(R_1 + R_2 - \epsilon_n) &\stackrel{(a)}{\leq} I(W_1; Y_1^n, S^n) + I(W_2; Y_2^n, S^n) \\
&\stackrel{(b)}{\leq} I(W_1; Y_1^n | S^n) + I(W_2; Y_1^n, Y_2^n, Y_R^n | S^n, W_1) \\
&\stackrel{(c)}{\leq} \sum h(Y_{1t} | S^n, Y_1^{t-1}) \\
&\quad - \sum h(Y_{1t} | S^n, W_1, W_2, Y_1^{t-1}, Y_2^{t-1}, Y_R^{t-1}) \\
&\quad + \sum h(Y_{2t}, Y_{Rt} | S^n, W_1, Y_1^{t-1}, Y_2^{t-1}, Y_R^{t-1}, Y_{1t}) \\
&\quad - \sum h(Y_{2t}, Y_{Rt} | S^n, W_1, W_2, Y_1^{t-1}, Y_2^{t-1}, Y_R^{t-1}, Y_{1t}) \\
&\stackrel{(d)}{\leq} \sum h(Y_{1t} | S_t) - \sum h(Z_{1t}) \\
&\quad + \sum h(Y_{2t}, Y_{Rt} | S_t, X_{1t}, X_{Rt}, Y_{1t}) - \sum h(Z_{2t}, Z_{Rt}) \\
&\stackrel{(e)}{\leq} p^2 \sum h(\mathbf{H}_{11} X_{1t} + \mathbf{H}_{12} X_{2t} + \mathbf{H}_{1R} X_{Rt} + Z_{1t}) \\
&\quad + p(1-p) \sum h(\mathbf{H}_{11} X_{1t} + \mathbf{H}_{1R} X_{Rt} + Z_{1t}) \\
&\quad + (1-p)p \sum h(\mathbf{H}_{12} X_{2t} + \mathbf{H}_{1R} X_{Rt} + Z_{1t}) \\
&\quad + (1-p)^2 \sum h(\mathbf{H}_{1R} X_{Rt} + Z_{1t}) \\
&\quad - \sum h(Z_{1t}) \\
&\quad + p \sum h(\mathbf{H}_{22} X_{2t} + Z_{2t}, \mathbf{H}_{R2} X_{2t} + Z_{Rt} | \mathbf{H}_{12} X_{2t} + Z_{1t}) \\
&\quad - p \sum h(Z_{2t}, Z_{Rt}),
\end{aligned}$$

where (a) is from Fano's inequality; (b) is from the mutual independence of  $(W_1, W_2, S^n)$ ; (c) is from conditioning reduces entropy; (d) is from  $X_{kt} = f_{kt}(W_k, S^{t-1})$  and

$X_{Rt} = f_{Rt}(Y_R^{t-1}, S^t)$ , the mutual independence of  $(Z_1^n, Z_2^n, Z_R^n, W_1, W_2, S^n)$ , the i.i.d. assumption of  $(Z_1^n, Z_2^n, Z_R^n)$ , and conditioning reduces entropy; (e) is from conditioning reduces entropy, and the evaluation of  $S_t$ .

To get the claimed outer bound on  $d_1 + d_2$ , we evaluate the above bound with the Gaussian distributions that maximize the differential entropies [9], and take the limit as  $P \rightarrow \infty$ .

## VI. CONCLUSION

We found that an in-band relay can provide a DoF gain in the two-user *bursty* MIMO Gaussian IC. More importantly, we showed that the relay can help achieve *interference-free* DoF performances for certain antenna configurations. The relay and the transmitters cooperate by exploiting information of the bursty traffic states to achieve the performances. We observed that the gain can be particularly substantial with low data traffic, as it can scale *linearly* with the number of antennas at the relay. Our results show promising benefits of relays in practical wireless systems where multiple source-destination links interfere with each other in a bursty manner due to intermittent data traffic.

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