Preamble Design for Channel Estimation in MIMO-OFDM Systems

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***Abstract*— A maximum-likelihood (ML) channel estimation and preamble design rules for the multiple-input-multiple-output (MIMO) channels are described when orthogonal frequency division multiplexing (OFDM) is employed with null subcarriers at both DC and high frequencies. The ML channel estimator in the time domain is derived assuming the knowledge of the maximum length of channel. To reduce the mean square error (MSE) of the estimation, three design rules are proposed: orthogonality between the preambles of different antennas, orthogonality between the circular shifted preamble sequences, and the condition that the number of subcarriers is larger than the maximum length of channel multiplied by the number of transmit antennas. In addition, we prove that the use of Golay Complementary Sequence (GCS) for preamble limits the peak to average power ratio (PAPR) by 3dB although there are null subcarriers at high frequencies. Numerical results shows that the MSE of the proposed method approaches that of optimum ML estimation when the number of null subcarriers and maximum length of channel are small.**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) systems have been applied to the wireless communications due to their robustness to multipath fading and the high bandwidth efficiency. Multiple transmit and receive antennas are combined with OFDM to improve the capacity and reliability of communications [1]. As multiple-input-multiple-output (MIMO) OFDM systems exploit diversity combining and coherent detection, channel estimation becomes more important as the number of antennas increases.

In single-input-single-output (SISO) OFDM systems, the least square (LS) estimation in the frequency domain is generally used for the preamble-based channel estimations. Although the complexity of the LS estimation is small, the mean square error (MSE) of the estimation is relatively high. In order to reduce the MSE, the time-domain ML channel estimation (TCE) is considered in [4] for SISO-OFDM systems. In MIMO-OFDM systems, the minimum mean square error (MMSE) channel estimation in the frequency domain is considered in [2], which requires the channel statistics. In practical OFDM systems, DC subcarrier is not used due to DC offset problem and subcarriers at high frequencies are not used to avoid adjacent channel interference [5],[6]. In [3], the effect of null subcarriers to the frequency-domain channel estimation is considered. In addition, the peak to average power ratio (PAPR) of the preamble sequence should be kept small to avoid the non linear distortions, which is considered in [7] by using Golay Complementary Sequence (GCS) for the preamble sequence [8],[9].

For the frequency-domain LS channel estimation of MIMO-OFDM systems, the preamble sequence of each transmit antenna should use a different set of subcarriers. Thus, the MSE of the frequency-domain LS channel estimation increases significantly as the number of transmit antenna increases because the estimated channel coefficients are interpolated in the frequency domain. To avoid the interpolation, the estimation duration is increased. On the other hand, the time-domain ML channel estimator (TCE) does not suffer from the interpolation error, so it can reduce the estimation duration.

In this paper, a maximum-likelihood (ML) channel estimation and preamble design rules are described when MIMO-OFDM is employed with null subcarriers at both DC and high frequencies. To reduce the MSE of the estimation, three design rules are proposed: orthogonality between the preambles of different antennas, orthogonality between the circular shifted preamble sequences, and the condition that the number of subcarriers is larger than the maximum length of channel multiplied by the number of transmit antennas. In addition, we prove that the use of Golay Complementary Sequence (GCS) for preamble limits the peak to average power ratio (PAPR) by 3 dB although there are null subcarriers at high frequencies. Numerical results shows that the MSE of the proposed method approaches that of optimum ML estimation when the number of null subcarriers and maximum length of channel are small.

II. SIGNAL MODEL

MIMO-OFDM systems considered in this paper are shown in Fig. 1. At each transmit antenna, the data is modulated by inverse fast Fourier transform (IFFT) and a cyclic prefix (CP) of length ν is added. To avoid intersymbol interference (ISI), $\nu > L-1$, where L is the maximum length of each channel. The system has N_T transmit antennas and N_R receive antennas. We assume a frequency-selective channel, which remains unchanged during at least one OFDM symbol.

Using the preamble sequence X_k^t in the frequency domain, the preamble sequence is allocated at subcarriers where the subcarrier index is the component of the position set $\mathbf{p}^t$. In other words, the position set $\mathbf{p}^t$ of the t^{th} transmit antenna indicates the location of the preamble sequences. Given N_T transmit antennas, the time-domain transmit signal at the t^{th} transmit antenna is expressed as

$$\begin{aligned} x_n^t &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k^t \exp\left(j\frac{2\pi nk}{N}\right) \\ X_k^t &= \begin{cases} \text{nonzero} & k \in \mathbf{p}^t \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{1}$$

Here, the index, k denotes the k^{th} subcarrier.

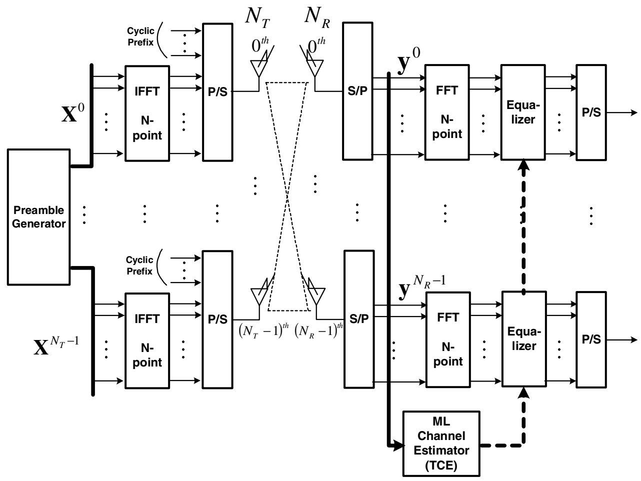

Fig. 1. MIMO-OFDM Systems

Given N_R receive antennas, the time-domain received signal at the r^{th} receive antenna after removing the cyclic prefix is given by

$$y_n^r = \sum_{t=0}^{N_T-1} \sum_{l=0}^{L-1} x^t_{(n-l)_{mod\,N}} h_l^{tr} + \eta_n^r, \quad 0 \le n \le N-1 \quad (2)$$

where h_l^{tr} is the channel parameter between the t^{th} transmit antenna and the r^{th} receive antenna. The indices n and l indicate time and channel, respectively. N is the preamble length and equal to the IFFT size because we assume that the estimation duration is one OFDM symbol. η_n^r is assumed to be additive white Gaussian noise (AWGN).

Eq. (2) can be written in matrix form as

$$\mathbf{y} = \mathbf{x}\mathbf{h} + \boldsymbol{\eta} \quad (3)$$

where $\mathbf{y} = [\mathbf{y}^0\ \mathbf{y}^1\ \cdots\ \mathbf{y}^{N_R-1}]$, $\mathbf{y}^r = [y_0^r\ y_1^r\ \cdots\ y_{N-1}^r]^T$ and the superscript $[\cdot]^T$ indicates transpose. $\mathbf{x}$ is a $N \times LN_T$ matrix

$$\mathbf{x} = \left[\mathbf{x}^0\ \mathbf{x}^1\ \cdots\ \mathbf{x}^{N_T-1}\right] \quad (4)$$

$$\mathbf{x}^t = \begin{bmatrix} x_0^t & x_{N-1}^t & \cdots & x_{N-L+1}^t \\ x_1^t & x_0^t & \cdots & x_{N-L+2}^t \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-1}^t & x_{N-2}^t & \cdots & x_{N-L}^t \end{bmatrix} \quad (5)$$

and $\mathbf{h}$ is the time-domain channel matrix with dimension, $LN_T \times N_R$

$$\mathbf{h} = \begin{bmatrix} \mathbf{h}^{00} & \mathbf{h}^{01} & \cdots & \mathbf{h}^{0(N_R-1)} \\ \mathbf{h}^{10} & \mathbf{h}^{11} & \cdots & \mathbf{h}^{1(N_R-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{h}^{(N_T-1)0} & \mathbf{h}^{(N_T-1)1} & \cdots & \mathbf{h}^{(N_T-1)(N_R-1)} \end{bmatrix} \quad (6)$$

where $\mathbf{h}^{tr} = [h_0^{tr}\ \ h_1^{tr}\ \ \cdots\ \ h_{L-1}^{tr}]^T$. Finally, $\boldsymbol{\eta} = [\boldsymbol{\eta}^0\ \ \boldsymbol{\eta}^1\ \ \cdots\ \ \boldsymbol{\eta}^{N_R-1}]$ and $\boldsymbol{\eta}^r = [\eta_0^r\ \ \eta_1^r\ \ \cdots\ \ \eta_{N-1}^r]^T$ is a zero-mean white Gaussian vector with covariance matrix $\mathbf{C}_{\boldsymbol{\eta}^r} = E\{\boldsymbol{\eta}^r \boldsymbol{\eta}^{r\,H}\} = \sigma_\eta^2 \mathbf{I}_N$ where I_N is the $N \times N$ identity matrix and the superscript $[\cdot]^H$ indicates Hermitian transpose.

In the frequency domain, channel matrix with dimension, $NN_T \times N_R$ is defined as

$$\mathbf{H} \equiv \begin{bmatrix} \mathbf{H}^{00} & \mathbf{H}^{01} & \cdots & \mathbf{H}^{0(N_R-1)} \\ \mathbf{H}^{10} & \mathbf{H}^{11} & \cdots & \mathbf{H}^{1(N_R-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}^{(N_T-1)0} & \mathbf{H}^{(N_T-1)1} & \cdots & \mathbf{H}^{(N_T-1)(N_R-1)} \end{bmatrix} \quad (7)$$

where $\mathbf{H}^{tr} = [H_0^{tr}\ H_1^{tr}\ \cdots\ H_{N-1}^{tr}]^T$ is the frequency channel vector between the t^{th} transmit antenna and the r^{th} receive antenna. The relation between $\mathbf{h}^{tr}$ and $\mathbf{H}^{tr}$ is

$$\mathbf{H}^{tr} = \mathbf{W}_L \mathbf{h}^{tr} \quad (8)$$

where $[\mathbf{W}_L]_{p,q} = \exp\left(-j\frac{2\pi pq}{N}\right)$, $(N \times L)$.

III. ML Channel Estimator

A. Derivation of ML Channel Estimator

Given $\mathbf{h}$, the conditional probability density function is given by

$$\Lambda(\mathbf{y}|\mathbf{h}) = \frac{1}{\left(\pi\sigma_\eta^2\right)^{NN_R}} \exp\left\{-\frac{1}{\sigma_\eta^2} Tr\left\{[\mathbf{y} - \mathbf{x}\mathbf{h}]^H[\mathbf{y} - \mathbf{x}\mathbf{h}]\right\}\right\} \quad (9)$$

where $Tr\{\cdot\}$ denotes the trace of a square matrix. The ML estimation of $\mathbf{h}$ can be found by maximizing eq. (9). This is equivalent to minimizing the log likelihood function:

$$\Lambda_L(\mathbf{y}|\mathbf{h}) = Tr\left\{[\mathbf{y} - \mathbf{x}\mathbf{h}]^H[\mathbf{y} - \mathbf{x}\mathbf{h}]\right\}. \quad (10)$$

Since $\Lambda_L(\mathbf{y}|\mathbf{h})$ is a convex function over $\mathbf{h}$, the estimation can be obtained by choosing $\mathbf{h}$ that satisfies the following condition

$$\frac{\partial \Lambda_L(\mathbf{y}|\mathbf{h})}{\partial \mathbf{h}} = 0. \quad (11)$$

This condition implies that the ML channel estimate is given by

$$\hat{\mathbf{h}} = \left(\mathbf{x}^H\mathbf{x}\right)^{-1}\mathbf{x}^H\mathbf{y}. \quad (12)$$

B. Performance of ML Channel Estimator

In this subsection, the MSE of the time-domain ML channel estimator (TCE) is derived as the performance measure. Since the channel equalization is performed in the frequency domain, the MSE of TCE is converted into that of the frequency-domain channel. Then the preamble condition that minimizes the MSE is derived.

From eq. (3) and (12), the MSE of TCE is given by

$$\begin{aligned} \text{MSE}_{h,\text{TCE}} &= \frac{1}{LN_TN_R} E\left[Tr\left\{\left(\hat{\mathbf{h}} - \mathbf{h}\right)^H\left(\hat{\mathbf{h}} - \mathbf{h}\right)\right\}\right] \\ &= \frac{1}{LN_TN_R} E\left[Tr\left\{\left(\mathbf{x}^H\mathbf{x}\right)^{-1}\mathbf{x}^H\boldsymbol{\eta}\boldsymbol{\eta}^H\mathbf{x}\left(\mathbf{x}^H\mathbf{x}\right)^{-1}\right\}\right] \\ &= \frac{\sigma_\eta^2}{LN_T} Tr\left\{\left(\mathbf{x}^H\mathbf{x}\right)^{-1}\right\} \end{aligned} \quad (13)$$

Using eq. (7), (8) and (13), the MSE of TCE over frequency-domain channel is given by

$$\begin{aligned}\mathrm{MSE}_{H,\mathrm{TCE}} &= \frac{1}{NN_TN_R}E\left[Tr\left\{\left(\hat{\mathbf{H}}-\mathbf{H}\right)^H\left(\hat{\mathbf{H}}-\mathbf{H}\right)\right\}\right]\\ &= \frac{1}{N_TN_R}E\left[\sum_{t=0}^{N_T-1}\sum_{r=0}^{N_R-1}Tr\left\{\left(\hat{\mathbf{h}}^{tr}-\mathbf{h}^{tr}\right)^H\left(\hat{\mathbf{h}}^{tr}-\mathbf{h}^{tr}\right)\right\}\right]\\ &= \frac{\sigma_\eta^2}{N_T}\sum_{t=0}^{N_T-1}Tr\left\{\left(\mathbf{x}^{t^H}\mathbf{x}^t\right)^{-1}\right\}\end{aligned} \tag{14}$$

In [4], the optimum preamble condition was derived for TCE in SISO systems. Similarly, in MIMO-OFDM systems, the optimum preamble condition to minimize the MSE (13) or (14) is

$$\mathbf{x}^H\mathbf{x} = \frac{N}{N_T}\mathbf{I}_{LN_T} \tag{15}$$

if the sum of power of the preamble sequence in each transmit antenna is constant (i.e., $\sum_{k=0}^{N-1}|X_k^t|^2 = N/N_T,\ \ \forall t \in \{0,1,\cdots,N_T-1\}$). The preamble condition (15) is the extension of the preamble condition in [5] for SISO-OFDM. Employing the optimum preamble condition (15), the MSE of TCE is given by

$$\mathrm{MSE}_{h,\mathrm{TCE}}^{opt} = \frac{N_T}{N}\sigma_\eta^2 \tag{16}$$

$$\mathrm{MSE}_{H,\mathrm{TCE}}^{opt} = \frac{LN_T}{N}\sigma_\eta^2 \tag{17}$$

As the number of transmit antenna increases, both $\mathrm{MSE}_{h,\mathrm{TCE}}^{opt}$ and $\mathrm{MSE}_{H,\mathrm{TCE}}^{opt}$ increase because total transmit power is assumed to be the same regardless of the number of the transmit antennas. Note that $\mathrm{MSE}_{H,\mathrm{TCE}}^{opt}$ is L times larger than $\mathrm{MSE}_{h,\mathrm{TCE}}^{opt}$ because the noise is increased L times through FFT.

IV. Preamble Design Rule

In this section, we develop the optimum preamble criteria for time-domain ML channel estimation (TCE) in MIMO-OFDM considering null subcarriers in DC and high frequencies. The preamble design rule is derived considering two optimization criteria: minimizing MSE of TCE (17), and reducing the PAPR of the transmit preamble sequence. Then, an example of preamble design according to the proposed rule is presented for a case of four transmit antennas.

The optimum preamble condition (15) imposes the orthogonality between any of two columns selected from $\mathbf{x}$:

$$\mathbf{x}^H\mathbf{x} = \frac{N}{N_T}\mathbf{I}_{LN_T} \iff \mathbf{x}_i^{t^H}\mathbf{x}_j^r = \begin{cases} N/N_T & (t=r)\ \&\ (i=j) \\ 0 & \text{otherwise}\end{cases} \tag{18}$$

where $\mathbf{x}_i^t$ is the $((t+1)\times(i+1))^{th}$ column vector of $\mathbf{x}$, indicating the i^{th} circular shifted sequence of the t^{th} transmit antenna in the time domain. In practice, this condition imposes the orthogonality between not only antennas but also the delayed sequences in each transmit antenna. By allocating different subcarriers to different antennas, the orthogonality between antennas can be obtained. Specifically, the orthogonality is satisfied if the preamble of transmit antenna is mapped at subcarriers following the equi-spaced position set:

$$\mathbf{p}_{eq}^t \equiv \{t, t+N_T, \cdots, t+(N/N_T-1)N_T\},\quad \forall t \in \{0,1,\cdots,N_T-1\} \tag{19}$$

For the orthogonality between the circular shifted sequences, the preamble components of each transmit antenna should have the same magnitude and the preamble length N should be greater than LN_T. These properties are summarized in Theorem 1, which will be proved in Appendix I. In OFDM systems, it is not difficult to overcome the limitation about N because the maximum length of channel, L is shorter than the cyclic prefix, which is commonly shorter than $N/4$ [5],[6].

Theorem 1: Let $\mathbf{X}^t$ be the preamble sequence of the t^{th} transmit antenna mapped at the equi-spaced position set $\mathbf{p}_{eq}^t$. N denotes the IFFT size and the number of transmit antenna N_T is the same as the spacing in the equi-spaced position set. The number of preamble components at $\mathbf{p}_{eq}^t$ becomes N/N_T, and the transmit sequence in the frequency domain is

$$\mathbf{X}^t = [\cdots\ X_t^t\ \ \mathbf{0}\ \ X_{t+N_T}^t\ \ \mathbf{0}\ \ \cdots\ \ \mathbf{0}\ \ X_{t+N-N_T}^t\ \ \cdots]^T \tag{20}$$

where $\mathbf{0} = [0\ \cdots\ 0],\ 1\times(N_T-1)$. Let $\mathbf{x}_0^t$ be the time-domain sequence when $\mathbf{X}^t$ is the IFFT input and $\mathbf{x}_i^t$ be the i^{th} circular shifted sequence of $\mathbf{x}_0^t$. Then,

$$\left\{\begin{array}{l} |X_k^t| = 1\ (\text{constant}), \forall k \in \mathbf{p}_{eq}^t \\ N > L\times N_T \end{array}\right\} \iff \mathbf{x}_i^{t^H}\mathbf{x}_j^t = \begin{cases} N/N_T & i=j \\ 0 & \text{otherwise.}\end{cases} \tag{21}$$ ■

In practical OFDM systems, the subcarriers at DC or high frequencies [5],[6] cannot be used because DC subcarrier is not used due to DC offset problem and subcarriers at high frequencies increases adjacent channel interference. In other words, the subcarriers that are the components of the position set $\mathbf{p}_{null}$ are not used where $\mathbf{p}_{null}$ is defined as follows:

$$\mathbf{p}_{null} \equiv \{\underbrace{0}_{\text{DC}}, \underbrace{N_u/2+1, N_u/2+2, \cdots, N-1-N_u/2}_{\text{High Frequencies}}\} \tag{22}$$

where N_u denotes the number of used subcarriers. Due to the null subcarriers, the orthogonality between the circular shifted sequences cannot be maintained because the condition in Theorem 1 is sufficient and necessary. As a suboptimal rule, we allocate the preamble sequences to the subcarriers which belongs to $\mathbf{p}_{eq}^t$ that are not either DC or high frequencies. In other words, the allocated subcarriers which belongs to the position set $\left(\mathbf{p}_{eq}^t - \mathbf{p}_{null}\right)$ are employed for transmitting the preamble sequence. The MSE increase of the preamble design is negligible when L and null subcarriers are small, which will be evaluated numerically.

In practical OFDM systems, a low PAPR sequence should be used as the preamble. In [7], it is shown that GCS can be used to construct the sequence with PAPR less or equal to 3 dB. In this paper, we propose that GCS can be used as the preamble of all the transmit antennas except the 0^{th} transmit antenna even though null subcarriers are used at DC and high frequencies. This is summarized in Theorem

2, which will be proved in Appendix II. According to the proof of Theorem 2, GCS at the equi-spaced positions can have a low PAPR ($\leq 3dB$). But the preamble of the 0^{th} transmit antenna cannot be mapped at equi-spaced positions due to the null at DC subcarrier. Therefore, GCS cannot be used as the preamble of the 0^{th} transmit antenna. For the 0^{th} transmit antenna, any low PAPR sequence which is not GCS can be used as the preamble. There exists many GCSs with certain length, $2^{\alpha}10^{\beta}26^{\gamma}(\alpha, \beta, \gamma \geq 0)$ [9]. In [8] and [9], GCS with length $2AB$ or AB can be constructed from Golay Complementary Pairs with length A and B. Therefore, the lowest-PAPR GCS can be used as the preamble sequence among many GCSs.

Theorem 2: Let $\mathbf{a}, \mathbf{b}$ be the equi-powered Golay Complementary Pair with length M which is assumed to be equal to the number of all the elements of $(\mathbf{p}_{eq}^{t} - \mathbf{p}_{null})$.

$$\begin{aligned} \mathbf{a} &= [a_0, a_1, \cdots, a_{M-1}]^T \\ \mathbf{b} &= [b_0, b_1, \cdots, b_{M-1}]^T \end{aligned} \tag{23}$$

$\mathbf{c}$ is the IFFT-input vector, composed of $\mathbf{a}$ at $(\mathbf{p}_{eq}^{t} - \mathbf{p}_{null})$ and null subcarriers in other positions where $1 \leq t \leq N_T - 1$. Similarly, $\mathbf{d}$ can be generated from $\mathbf{b}$.

$$\begin{aligned} \mathbf{c} = [\text{DC} \ \cdots \ a_{M/2} \ \mathbf{0} \ \cdots \ \mathbf{0} \ a_{M-1} \ \cdots \\ \text{High} \cdots \ a_0 \ \mathbf{0} \ \cdots \ \mathbf{0} \ a_{M/2-1} \ \cdots]^T \\ \mathbf{d} = [\text{DC} \ \cdots \ b_{M/2} \ \mathbf{0} \ \cdots \ \mathbf{0} \ b_{M-1} \ \cdots \\ \text{High} \ \cdots \ b_0 \ \mathbf{0} \ \cdots \ \mathbf{0} \ b_{M/2-1} \ \cdots]^T \end{aligned} \tag{24}$$

where $\mathbf{0} = [0 \ \cdots \ 0]$, $1 \times (N_T - 1)$. Let $\mathbf{e}$ and $\mathbf{f}$ be the IFFT-output vector when $\mathbf{c}$ and $\mathbf{d}$ are the IFFT inputs, respectively. Then,

$$\text{PAPR}[\mathbf{e}, \mathbf{f}] \leq 2.$$

PAPR is defined as

$$\text{PAPR}[\mathbf{e}] = \frac{\max\{P_n^{\mathbf{e}}\}}{E[P_n^{\mathbf{e}}|n]}$$

where $P_n^{\mathbf{e}}$ is the instantaneous power of e_n. ■

Summarizing the properties, we can obtain the preamble design rule as follows:

Preamble Design Rule: *The preamble sequence in the t^{th} transmit antenna consists of the equi-powered GCS at $(\mathbf{p}_{eq}^{t} - \mathbf{p}_{null})$ and null subcarriers in other positions. The preamble length N should be greater than LN_T and we use GCS which has the lowest PAPR among many GCSs. For the 0^{th} transmit antenna, the preamble sequence can be found through the exhaustive search considering a low PAPR as the criterion.*

$$\begin{aligned} \{X_k^t | k \in (\mathbf{p}_{eq}^{t} - \mathbf{p}_{null})\} &: Golay\ Complementary\ Sequence, \\ \forall t \in \{1, 2, \cdots, N_T - 1\}, &\quad subject\ to\ N > LN_T \end{aligned} \tag{25}$$

An specific example is shown using the above rule. It is assumed that the IFFT size $N = 64$, the number of transmit antenna $N_T = 4$ and the number of used subcarriers $N_u = 40$.

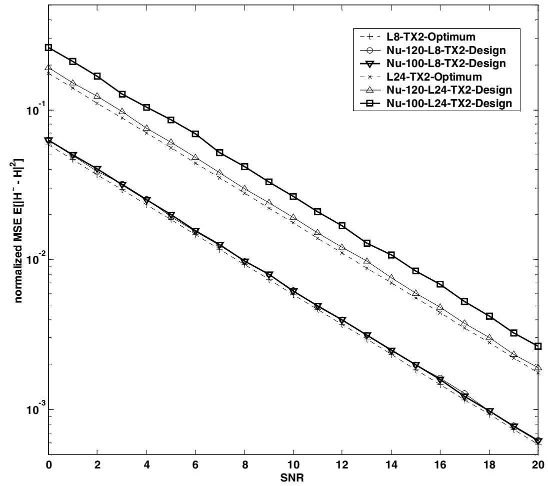

Fig. 2. MSE of optimum bound and preamble design

At first, the position set in each transmit antenna is given by

$$\begin{aligned} \mathbf{p}_{eq}^{0} - \mathbf{p}_{null} &= \{4, 8, \cdots, 20, 44(-20), 48(-16), \cdots, 60(-4)\} \\ \mathbf{p}_{eq}^{1} - \mathbf{p}_{null} &= \{1, 5, \cdots, 17, 45(-19), 49(-15), \cdots, 61(-3)\} \\ \mathbf{p}_{eq}^{2} - \mathbf{p}_{null} &= \{2, 6, \cdots, 18, 46(-18), 50(-14), \cdots, 62(-2)\} \\ \mathbf{p}_{eq}^{3} - \mathbf{p}_{null} &= \{3, 7, \cdots, 19, 47(-17), 51(-13), \cdots, 63(-1)\} \\ \mathbf{p}_{null} &= \{21, 22, \cdots, 31, 32(-32), 33(-31), \cdots, 43(-21)\} \end{aligned} \tag{26}$$

Considering the equi-powered preamble condition, binary phase shift keying (BPSK) signaling can be used. According to the constraint about N, the maximum length of channel L should be less than 16. Specifically, GCS with length 10 is used as the preamble components at $(\mathbf{p}_{eq}^{t} - \mathbf{p}_{null})$ except $t = 0$.

$$\begin{aligned} \{X_k^t | k \in (\mathbf{p}_{eq}^{t} - \mathbf{p}_{null})\} &= \{G_5^t, \cdots, G_9^t, G_0^t, \cdots, G_4^t\} \\ \{G_k^t | k = 0, 1, \cdots, 9\} &= \{1, 1, 1, 1, 1, -1, 1, -1, -1, 1\} \\ &\forall t \in \{1, 2, \cdots, N_T - 1\} \end{aligned} \tag{27}$$

The preamble sequence achieves a low PAPR of 2.489 dB. Through the exhaustive search, the preamble sequence of the 0^{th} transmit antenna is found. The sequence with a low PAPR of 2.041 dB is given by

$$\{G_k^0 | k = 0, 1, \cdots, 9\} = \{1, 1, 1, 1, -1, 1, 1, -1, 1, -1\} \tag{28}$$

Although the preamble sequence of the 0^{th} transmit antenna is used as that of the t^{th} transmit antenna, a low PAPR of 2.041 dB cannot be obtained since the preamble sequence of the 0^{th} transmit antenna does not have the equi-spaced positions. In this example, fortunately, the preamble sequence of the 0^{th} transmit antenna with a low PAPR can be easily found because the number of preamble components is small. But, the search time will be exponentially increased as the number of preamble components is large. On the other hand, GCS can be easily found using the generation rule [8],[9] even if the length is large.

V. Numerical Results

Fig. 2 shows the MSE of the proposed preamble design using 100 or 120 subcarriers out of 128 available subcarriers, i.e., $N_u = 100$ or 120, and $N = 128$. The number of transmit antenna N_T is two, and the number of receive antenna N_R is one. The channel is assumed to be rayleigh fading without correlation in antennas and have the equal gain profile. The dashed line indicates the MSE of the optimum bound (17) and the solid line denotes the MSE of preamble design. From Fig. 2, the MSE of the preamble design is near optimum (< 0.5 dB) in $N_u = 120$, $N_T = 2$ and $L \leq 24$. In addition, as N_u decreases and L increases, performance degradation becomes larger. The performance depends on N_u, N_T and L. When N_u equals to N, the MSE of preamble design is exactly the same with the optimum bound regardless of N_T and L. However, as N_u decreases, the degradation of the MSE becomes larger as L or N_T increases.

VI. Conclusions

In this paper, a ML channel estimator in the time-domain was derived in MIMO-OFDM systems. Considering the MSE, null subcarriers at both DC and high frequencies, and a low PAPR, the preamble design rule was proposed. For a low PAPR, it was proved that the use of GCS for preamble limits the PAPR by 3 dB although there are null subcarriers at high frequencies. Numerical results have shown that the MSE of the proposed method approaches that of optimum ML estimation when $(N - N_u)$ and L are small.

Appendix I
Proof of Theorem 1

The i^{th} circular shifted sequence $\mathbf{x}_i^t$ is given by

$$\mathbf{x}_i^t = \mathbf{W}\mathbf{S}_i\mathbf{X}^t \tag{29}$$

where $\mathbf{W}$ is a normalized IFFT matrix and $\mathbf{S}_i = diag\left(1, e^{-j\frac{2\pi i}{N}}, \cdots, e^{-j\frac{2\pi(N-1)i}{N}}\right)$. Then, ${\mathbf{x}_i^t}^H\mathbf{x}_j^t$ is

$$\begin{aligned}
{\mathbf{x}_i^t}^H\mathbf{x}_j^t &= \left(\mathbf{W}\mathbf{S}_i\mathbf{X}^t\right)^H \mathbf{W}\mathbf{S}_j\mathbf{X}^t \\
&= \sum_{p=0}^{N/N_T-1} \left|X_{t+pN_T}^t\right|^2 e^{-j\frac{2\pi p(j-i)}{N/N_T}} \\
&= \begin{cases} \sum_{p=0}^{N/N_T-1} \left|X_{t+pN_T}^t\right|^2, & i = j \\ \sum_{p=0}^{N/N_T-1} \left|X_{t+pN_T}^t\right|^2, & |i-j| = q(N/N_T) \\ \sum_{p=0}^{N/N_T-1} \left|X_{t+pN_T}^t\right|^2 e^{-j\frac{2\pi p(j-i)}{N/N_T}}, & \text{otherwise} \end{cases}
\end{aligned} \tag{30}$$

where $0 \leq i, j \leq L-1$ and q is a positive integer. For eq. (30) to become zero when $i \neq j$, the case of $|i-j| = q(N/N_T)$ should not occur and $\left|X_{t+pN_T}^t\right|$ should be constant for all p. To avoid the case of $|i-j| = q(N/N_T)$, N should be greater than LN_T. Therefore, the necessary and sufficient condition for ${\mathbf{x}_i^t}^H\mathbf{x}_j^t = 0$ when $i \neq j$ is that $|X_k^t|$ is constant for all $k \in \mathbf{p}_{eq}^t$ and N is greater than LN_T.

Appendix II
Proof of Theorem 2

In N-point IFFT, the index, k ($\geq N/2$) is equivalent to the index, $k - N$. Using this property, the n^{th} element of $\mathbf{e}$ can be written as

$$\begin{aligned}
e_n &= \frac{1}{\sqrt{N}} \sum_{p=0}^{M-1} a_p e^{j\frac{2\pi n}{N}(t-(M/2+p)N_T)} \\
&= \frac{e^{j\frac{2\pi n}{N}(t-MN_T/2)}}{\sqrt{N}} \sum_{p=0}^{M-1} a_p e^{-j\frac{2\pi n p N_T}{N}} \\
&\qquad \forall t \in \{1, 2, \cdots, N_T - 1\}
\end{aligned} \tag{31}$$

The instantenous power of e_n is given by

$$\begin{aligned}
P_n^{\mathbf{e}} &= \frac{1}{N} \sum_{p=0}^{M-1} \sum_{q=0}^{M-1} a_p a_q^* e^{-j\frac{2\pi n}{N}(p-q)N_T} \\
&= \frac{\sum_{p=0}^{M-1} |a_p|^2}{N} + \frac{1}{N} \sum_{u \neq 0} C_a(|u|) e^{-j\frac{2\pi n |u| N_T}{N}}
\end{aligned} \tag{32}$$

where $C_a(|u|)$ is an aperiodic autocorrelation function defined by

$$C_a(|u|) = \sum_{p=0}^{M-1-|u|} a_p a_{p+|u|}^*, \quad -M+1 \leq u \leq M-1 \tag{33}$$

Similarly, $P_n^{\mathbf{f}}$ is generated from $\mathbf{b}$. Using the definition of GCS $(C_a(|u|) + C_b(|u|) = 0)$, the sum of $P_n^{\mathbf{e}}$ and $P_n^{\mathbf{f}}$ becomes

$$P_n^{\mathbf{e}} + P_n^{\mathbf{f}} = \frac{\sum_{p=0}^{M-1} \left(|a_p|^2 + |b_p|^2\right)}{N} = \frac{2\sum_{p=0}^{M-1} |a_p|^2}{N} \tag{34}$$

Since $P_n^{\mathbf{f}} \geq 0$ and $P_n^{\mathbf{e}} \geq 0$, PAPR of $\mathbf{e}$ and $\mathbf{f}$ are given by

$$\begin{aligned}
\text{PAPR}[\mathbf{e}] &= \frac{\max\{P_n^{\mathbf{e}}\}}{E\left[P_n^{\mathbf{e}}|n\right]} \leq 2 \\
\text{PAPR}[\mathbf{f}] &= \frac{\max\{P_n^{\mathbf{f}}\}}{E\left[P_n^{\mathbf{f}}|n\right]} \leq 2
\end{aligned} \tag{35}$$

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