ISIT 2009, Seoul, Korea, June 28 - July 3, 2009

Symmetric Feedback Capacity of the Gaussian Interference Channel to Within One Bit

Changho Suh
Wireless Foundations, EECS Department
University of California at Berkeley
Email: chsuh@eecs.berkeley.edu

David Tse
Wireless Foundations, EECS Department
University of California at Berkeley
Email: dtse@eecs.berkeley.edu

***Abstract*— We characterize the symmetric capacity of the two-user Gaussian interference channel with *feedback* to within 1 bit/s/Hz. The result makes use of a deterministic model to provide insights into the Gaussian channel. We derive a new outer bound to show that a proposed scheme can achieve the symmetric capacity to within one bit for all channel parameters. One consequence of the result is that feedback provides *unbounded* gain, i.e., the gain becomes arbitrarily large for certain channel parameters. It is a surprising result because feedback has been so far known to provide no gain in memoryless point-to-point channels and only power gain (*bounded* gain) in the multiple access channels. The gain comes from using feedback to fully exploit the side information provided by the broadcast nature of the wireless medium.**

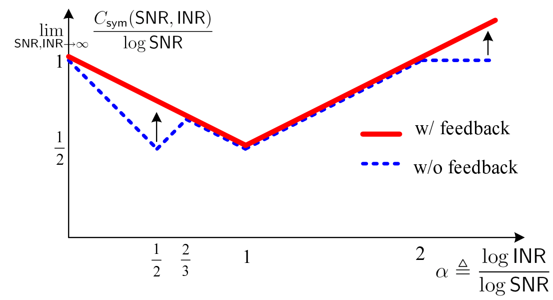

Fig. 1. The generalized degrees-of-freedom of the Gaussian interference channel with feedback

I. Introduction

Shannon showed that feedback does not increase capacity in the discrete-memoryless point-to-point channel [1]. However, in the multiple access channel (MAC), Gaarder and Wolf [2] showed that feedback could increase capacity although the channel is memoryless. Inspired by this result, Ozarow [3] found the feedback capacity region for the two-user Gaussian MAC. Ozarow's result implies that feedback provides only power gain (*bounded* gain). The reason of bounded gain is that transmitters cooperation induced by feedback can at most boost signal power (via aligning signal directions) in the MAC. Boosting signal power provides a capacity increase of a constant number of bits.

Now a question is "Will feedback help significantly in other channels where a receiver wants to decode *only desired* messages in the presence of *undesired* messages (interferences)?" To answer this question, we focus on the simple two-user Gaussian interference channel where each receiver wants to decode the messages only from its corresponding transmitter. We first characterize the symmetric feedback capacity for a *linear* deterministic model [4] well capturing key properties of the Gaussian channel. Gaining insights from this model, we develop a simple two-staged achievable scheme in the Gaussian channel. We then derive a new outer bound to show that the proposed scheme achieves the symmetric capacity to within one bit for all channel parameters.

An interesting consequence of this result is that feedback can provide *unbounded* gain in interference channels. This can be shown from the generalized degrees-of-freedom (g.d.o.f.) in Fig. 1, defined in [5] as $d(\alpha) \triangleq \lim_{\mathsf{SNR},\mathsf{INR}\to\infty} \frac{C_{\mathsf{sym}}(\mathsf{SNR},\mathsf{INR})}{\log \mathsf{SNR}}$, where α (x-axis) indicates the ratio of INR to SNR in dB scale: $\alpha \triangleq \frac{\log \mathsf{INR}}{\log \mathsf{SNR}}$. Notice that in certain weak interference regimes ($0 \le \alpha \le \frac{2}{3}$) and in the very strong interference regime ($\alpha \ge 2$), feedback gain becomes arbitrarily large as SNR and INR go to infinity. We will provide qualitative insights as to where this gain comes from.

Some work has been done in the interference channel with feedback [6], [7], [8], [9]. In [6], [7], Kramer developed a feedback strategy and derived an outer bound in the Gaussian channel. However, the gap between the outer bound and the inner bound becomes arbitrarily large with the increase of SNR and INR, for almost all cases except one specific point[1]. Recently, Jiang-Xin-Garg [9] found an achievable region in the discrete memoryless interference channel with feedback. However, their scheme employs three auxiliary random variables (requiring further optimization) and block Markov encoding (requiring a long block length). Also they did not provide any upper bounds. On the other hand, we propose a simple achievable scheme which is *explicit* and has *only two* stages. Also we derive a new tighter outer bound to characterize capacity to within 1 bit. Later we will provide more detailed comparison to Kramer's scheme [6] in Section IV-D.

This work was supported by Intel Corporation and the National Science Foundation under grant CNS-0722032.

[1]Although this strategy can be arbitrarily far from optimality, a careful analysis reveals that it can also provide unbounded feedback gain. See Fig. 4 for this.

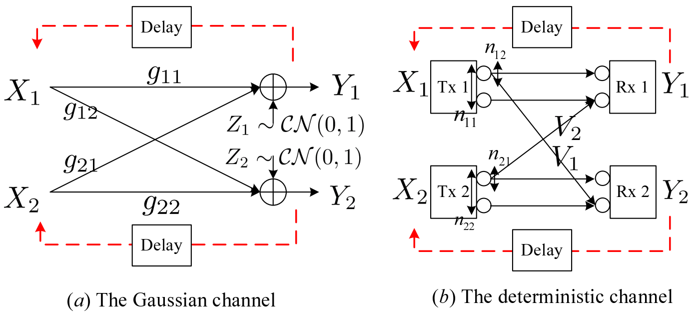

Fig. 2. Interference channels with feedback

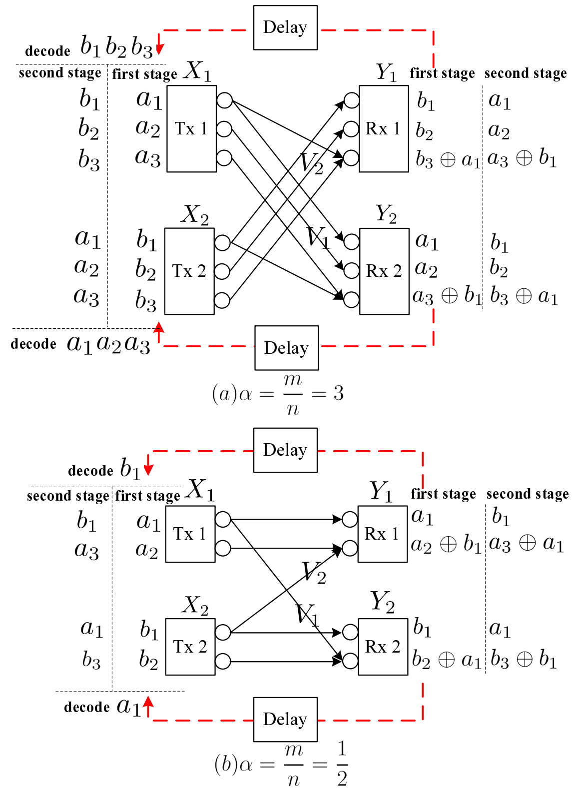

Fig. 3. An achievable scheme of the deterministic interference channel with feedback

II. Model

Fig. 2 (a) describes the Gaussian interference channel with feedback. We consider the symmetric interference channel where $g_{11} = g_{22} = g_d$, $g_{12} = g_{21} = g_c$, and $P_1 = P_2 = P$. Without loss of generality, we assume that signal power and noise power are normalized to 1, i.e., $P_k = 1$, $Z_k \sim \mathcal{CN}(0,1)$, $\forall k = 1,2$. Hence, signal-to-noise ratio and interference-to-noise ratio can be defined to capture channel gains: $\mathsf{SNR} \triangleq |g_d|^2$, $\mathsf{INR} \triangleq |g_c|^2$. There are two independent and uniformly distributed sources, $W_k \in \{1,2,\cdots,M_k\}, \forall k = 1,2$. Due to feedback, the encoded signal X_{ki} of user k at time i is a function of its own message and past output sequences: $X_{ki} = f_k^i\left(W_k, Y_k^{i-1}\right)$, where we use shorthand notation Y_k^{i-1} to indicate the sequence up to $i-1$. The symmetric capacity is defined by $C_{\mathsf{sym}} = \sup\{R : (R,R) \in \mathcal{C}\}$, where $\mathcal{C}$ is the capacity region.

III. The Deterministic Interference Channel

As a first step, we approximate the Gaussian channel by the deterministic model as shown in Fig. 2 (b). This model is useful because in the non-feedback case, the deterministic channel approximates the Gaussian channel to within a constant gap [10]. Our approach is to first come up with an optimal scheme for this model and then mimic it on the Gaussian channel.

The symmetric channel is characterized by two values: $n = n_{11} = n_{22}$ and $m = n_{12} = n_{21}$, where n and m indicate the number of signal bit levels that can be sent through direct link and cross link, respectively. Let V_k be a part of X_k *visible* to the other receiver. For each level, we assume a *modulo*-2-addition. Here, n and m correspond to channel gains (in dB scale) in the Gaussian channel: $n = \lfloor \log_2 \mathsf{SNR} \rfloor$ and $m = \lfloor \log_2 \mathsf{INR} \rfloor$.

Theorem 1: The symmetric feedback capacity of the deterministic interference channel is given by

$$C_{\mathsf{sym}} = \max\left(\frac{m}{2}, n - \frac{m}{2}\right). \quad (1)$$

A. Proof of Achievablility

Review of a non-feedback scheme [10]: Let us start by examining the schemes in the non-feedback case. The schemes are different depending on the strength of the interference. In the strong interference channel ($m \geq n$), the key observation is that all of the feasible rate tuples are decodable at both receivers: all messages are *common*. Since the maximum number of levels is m and maximally deliverable bits are limited by n, an achievable scheme is to send $\min(\frac{m}{2}, n)$. On the other hand, in the weak interference channel, information can be split into two parts: common m bits (*visible* to the other receiver); private $(n-m)$ bits (*invisible* to the other receiver). An achievable scheme is to send $(n-m)$ private bits on the lower levels (*invisible* to the other receiver) and some number of common bits on the upper levels. The number of common bits is decided depending on m and n.

The strong interference regime ($m \geq n$): Now let us go back to the feedback case. We explain the scheme with the simple example in Fig. 3 (a). Mimicking the non-feedback case, transmitters send only common information. The scheme uses two stages. In the first stage, transmitter 1 and 2 send (a_1, a_2, a_3) and (b_1, b_2, b_3), respectively. Each receiver defers decoding to the second stage. In the second stage, using feedback, each transmitter decodes information of the other user: transmitter 1 and 2 decode (b_1, b_2, b_3) and (a_1, a_2, a_3), respectively. Each transmitter then sends information of the other user. Now each receiver can decode its own data from two received signals. Notice that the second stage was used for refining all bits sent previously, without sending additional information. Therefore, the symmetric rate is $\frac{3}{2}$. Notice the improvement from the non-feedback rate of 1. We can easily extend the scheme to general (n,m). In the first stage, each transmitter sends m bits using all the levels. Using two stages, these m bits can be recovered with the help of feedback. Hence, we can achieve $R_{\sf sym} = \frac{m}{2}$.

The weak interference regime ($m < n$)**:** We explain the scheme with the simple example in Fig. 3 (b). Similar to the non-feedback case, information is split into two parts. In the first stage, transmitter 1 sends private information a_2 on the lower level (*invisible* to the other receiver) and common information a_1 on the upper signal level (*visible* to the other receiver). Similarly transmitter 2 sends b_1 and b_2. Receiver 1 then gets the clean signal a_1 on the upper level but gets the *interfered* signal $a_2 \oplus b_1$ on the lower level. Each receiver defers decoding to the second stage. In the second stage, with feedback, transmitter 1 and 2 can decode common information of the other user b_1 and a_1, respectively. Each transmitter then sends the other user information on the *upper* level; and sends new private information on the lower level. Now receiver 1 can decode the corrupted symbol a_2 sent in the first stage and also new private information a_3 by stripping a_1. During two stages, each receiver decodes three symbols during two time slots. Therefore, the symmetric rate is $\frac{3}{2}$. Notice the improvement from the non-feedback rate of 1, achieved by each user sending one bit at the lower level.

This scheme can be easily generalized to arbitrary (n, m). In the first stage, each transmitter sends m bits on the upper levels and $(n-m)$ bits on the lower levels. In the second stage, each transmitter forwards m bits of the other user on the upper levels and sends new $(n-m)$ private bits on the lower levels. Then, each receiver can decode all of the n bits sent in the first stage and new $(n-m)$ private bits sent in the second stage. Therefore, we can achieve $R_{\sf sym} = \frac{n+(n-m)}{2} = n - \frac{m}{2}$.

Remarks: There is feedback gain in both the strong and the weak regimes, but the nature of the gain is different. The gain in the strong interference regime is due to the fact that feedback provides a better *alternative* path through the two cross links . This concept coincides with *correlation routing* in [6]. On the other hand, in the weak interference channel, there is no better alternative path. In spite of this, it turned out that feedback gain could also be obtained in this regime. To understand this counterintuitive gain, let us look at the example of Fig. 3 (b) again. In the non-feedback case, every bit we get from the upper level, we have to pay \$2. This is because using the upper level causes the interference to the other receiver due to the *broadcast* nature of the wireless medium. This precludes us from using the top level for one user when we are already using the bottom level for the other user. On the other hand, if feedback is allowed, the *broadcast* nature can be *exploited*. We can see this from transmitter 1's sending of b_1 at time 2. This transmission allows receiver 1 to refine the corrupted symbol a_2 from $a_2 \oplus b_1$ *without* causing interference to receiver 2, since it already had the side information of b_1 from the previous broadcasting. We paid \$2 for the earlier transmission of b_1 at time 1, but now we can get a *rebate* of \$1. In other words, we exploited the *previous* broadcast transmission to reduce the cost of the broadcast transmission at time 2.

B. Proof of Converse

$$
\begin{aligned}
&N(R_1 + R_2 - \epsilon_N) \overset{(a)}{\leq} I(W_1; Y_1^N | W_2) + I(W_2; Y_2^N) \\
&\overset{(b)}{=} H(Y_1^N | W_2) + I(W_2; Y_2^N) \leq H(Y_1^N, V_1^N | W_2) + I(W_2; Y_2^N) \\
&\overset{(c)}{=} H(Y_1^N | V_1^N, W_2) + H(Y_2^N) \\
&\overset{(d)}{\leq} \sum \left[H(Y_{1i} | V_{1i}, V_{2i}) + H(Y_{2i}) \right]
\end{aligned}
$$

where (a) follows from the independence of (W_1, W_2) and Fano's inequality; (b) follows from the fact that Y_1^N is a function of W_1 and W_2; (c) follows from $H(V_1^N | W_2) = H(Y_2^N | W_2)$ (see Claim 1); (d) follows from the fact that X_2^N is a function of (W_2, V_1^{N-1}) (see Claim 2), V_2^N is a function of X_2^N; and conditioning reduces entropy.

Claim 1: $H(V_1^N | W_2) = H(Y_2^N | W_2)$.

Proof:

$$
\begin{aligned}
H(Y_2^N | W_2) &= \sum H(Y_{2i} | Y_2^{i-1}, W_2) \overset{(a)}{=} \sum H(V_{1i} | Y_2^{i-1}, W_2) \\
&\overset{(b)}{=} \sum H(V_{1i} | Y_2^{i-1}, W_2, X_2^i, V_1^{i-1}) \overset{(c)}{=} H(V_1^N | W_2),
\end{aligned}
$$

where (a) follows from the fact that Y_{2i} is a function of (X_{2i}, V_{1i}) and X_{2i} is a function of (W_2, Y_2^{i-1}); (b) follows from the fact that X_2^i is a function of (W_2, Y_2^{i-1}) and V_1^{i-1} is a function of (X_2^{i-1}, Y_2^{i-1}); (c) follows from the fact that Y_2^{i-1} is a function of (X_2^{i-1}, V_1^{i-1}) and X_2^i is a function of (W_2, V_1^{i-1}) (by Claim 2). ■

Claim 2: For all $i \geq 1$, X_1^i is a function of (W_1, V_2^{i-1}) and X_2^i is a function of (W_2, V_1^{i-1}).

Proof: By symmetry, it is enough to prove only one. Since the channel is deterministic (noiseless), X_1^i is a function of W_1 and W_2. In Fig. 2 (b), we can easily see that information of W_2 delivered to the first link must pass through V_{2i}. Also note that X_{1i} depends on the past output sequences until $i-1$ (due to feedback delay). Therefore, X_1^i is a function of (W_1, V_2^{i-1}). ■

If (R_1, R_2) is achievable, then $\epsilon_N \to 0$ as $N \to \infty$. Hence, we get: $R_1 + R_2 \leq H(Y_1 | V_1, V_2) + H(Y_2)$. We can easily show that the RHS is maximized when X_1 and X_2 are uniform and independent. Therefore, we get the desired result.

IV. The Gaussian Interference Channel

A. An Achievable Rate

Theorem 2: In the strong Gaussian interference channel ($\sf INR \geq SNR$), we can achieve

$$
R_{\sf sym}^{\sf strong} = \frac{1}{2} \log \left(1 + {\sf INR}\right). \tag{2}
$$

In the weak Gaussian interference channel ($\sf INR \leq SNR$), we can achieve

$$
R_{\sf sym}^{\sf weak} = \begin{cases} \log\left(1 + \frac{\sf SNR}{2{\sf INR}}\right) + \frac{1}{2}\log\left(1 + {\sf INR}\right) - \frac{1}{2}, & {\sf INR} \geq 1; \\ \log\left(1 + \frac{\sf SNR}{{\sf INR}+1}\right), & {\sf INR} \leq 1. \end{cases} \tag{3}
$$

Proof: **The strong interference regime** ($\sf INR \geq SNR$)**:** Mimicking the deterministic case, each transmitter sends only common information and employs two stages. In the first stage, each transmitter sends its own signal. In the second stage, each transmitter sends the information of the other user after decoding it with the help of feedback. In the Gaussian *noisy* channel, we need to be careful in how to combine the received signals (during two stages) to decode the message. Alamouti's scheme [11] gives insights into this. Notice that with feedback both messages are available at transmitters in the second stage. However, in spite of knowing both messages, transmitters cannot control the signals already sent in the first stage. Hence, they can only *partially* collaborate in the second stage. However, the beauty of Alamouti's scheme is that received signals can be designed to be *orthogonal* (for two time slots), although the signals in the first time slot are sent *without any coding*. This was well exploited and pointed out in distributed space-time codes [12]. Therefore, with Alamouti's scheme, transmitters are able to encode messages so that received signals are *orthogonal*. In the interference channel, orthogonality between different signals guarantees to completely remove the other signals (interference). This helps to improve performance significantly.

In the first stage (block), transmitter 1 and 2 send codewords X_1^N and X_2^N with rates R_1 and R_2, respectively. In the second stage, using feedback, transmitter 1 and 2 decode X_2^N and X_1^N, respectively. This can be decoded if $R_1, R_2 \leq \frac{1}{2}\log(1+\mathsf{INR})$ bits/s/Hz. Now we apply Alamouti's scheme. In the second stage, transmitter 1 sends X_2^{N*} and transmitter 2 sends $-X_1^{N*}$. Then, receiver 1 can gather two received signals: for $1 \leq i \leq N$,

$$\begin{bmatrix} Y_{1i}^{(1)} \\ Y_{1i}^{(2)*} \end{bmatrix} = \begin{bmatrix} g_d & g_c \\ -g_c^* & g_d^* \end{bmatrix} \begin{bmatrix} X_{1i} \\ X_{2i} \end{bmatrix} + \begin{bmatrix} Z_{1i}^{(1)} \\ Z_{1i}^{(2)*} \end{bmatrix}.$$

To extract X_{1i}, receiver 1 multiplies the row vector orthogonal to the vector corresponding to X_{2i}. Then, the codeword X_1^N can be decoded if $R_1 \leq \frac{1}{2}\log(1+\mathsf{SNR}+\mathsf{INR})$ bits/s/Hz. Similar operations are done at receiver 2. From all the rate constraints, we get the desired result (2).

The weak interference regime ($\mathsf{INR} \leq \mathsf{SNR}$)**:** Similar to the deterministic case, a scheme has two stages and information is split into common and private parts. Also recall that only common information is sent twice during two stages. Therefore, a natural idea is to apply Alamouti's scheme only for common information. Private information is newly sent for both stages.

In the first stage, transmitter 1 independently generates a common codeword X_{1c}^N and a private codeword $X_{1p}^{N,(1)}$ with rates R_{1c} and $R_{1p}^{(1)}$, respectively. For power splitting, we adapt the simplified Han-Kobayashi scheme [5] where private power is set such that private information is received at the noise level: $\lambda_p = \min\left(\frac{1}{\mathsf{INR}}, 1\right); \lambda_c = 1 - \lambda_p$. Now we assign power λ_p and λ_c to $X_{1p,i}^{(1)}$ and $X_{1c,i}$, $\forall i$, respectively; and *superpose* two signals to form channel input. Similarly transmitter 2 sends $X_{2p}^{N,(1)} + X_{2c}^N$. In the second stage, with feedback transmitter 1 and 2 decode X_{2c}^N and X_{2c}^N, respectively. This can be decoded if $R_{1c}, R_{2c} \leq \frac{1}{2}\log\left(1+\frac{\lambda_c \mathsf{INR}}{\lambda_p \mathsf{INR}+1}\right)$ bits/s/Hz. Now we apply Alamouti's scheme only for common information. Transmitter 1 sends X_{2c}^{N*} and just adds new private information $X_{1p}^{N,(2)}$. Transmitter 2 sends $-X_{1c}^{N*}$ and $X_{2p}^{N,(2)}$. Then, receiver 1 can gather the signals received during two stages. By the previous rate constraints for R_{1c} and R_{2c}, receiver 1 can decode X_{1c}^N and X_{2c}^N. Now each receiver decodes its private messages after subtracting common information: $R_{1p}^{(1)}, R_{2p}^{(1)} \leq \log\left(1+\frac{\lambda_p \mathsf{SNR}}{\lambda_p \mathsf{INR}+1}\right)$ bits/s/Hz. Under the simple power setting of λ_p and λ_c, we get the desired result (3). ■

B. An Outer Bound

Theorem 3: The symmetric capacity of the Gaussian interference channel with feedback is upper-bounded by

$$C_{\mathsf{sym}} \leq \frac{1}{2}\sup_{0\leq\rho\leq 1}\left[\log\left(1+\frac{(1-\rho^2)\mathsf{SNR}}{1+(1-\rho^2)\mathsf{INR}}\right) + \log\left(1+\mathsf{SNR}+\mathsf{INR}+2\rho\sqrt{\mathsf{SNR}\cdot\mathsf{INR}}\right)\right]. \quad (4)$$

Proof: For side information, we consider the interference plus noise: $S_1 = g_c X_1 + Z_2$. Using this, we get

$$\begin{aligned} N(R_1+R_2-\epsilon_N) &\overset{(a)}{\leq} I(W_1;Y_1^N,S_1^N|W_2) + I(W_2;Y_2^N) \\ &= h(Y_1^N,S_1^N|W_2) - h(Y_1^N,S_1^N|W_1,W_2) + I(W_2;Y_2^N) \\ &\overset{(b)}{=} h(Y_1^N,S_1^N|W_2) - \sum[h(Z_{1i})+h(Z_{2i})] + I(W_2;Y_2^N) \\ &\overset{(c)}{=} h(Y_1^N|S_1^N,W_2,X_2^N) - \sum h(Z_{1i}) + h(Y_2^N) - \sum h(Z_{2i}) \\ &\overset{(d)}{\leq} \sum[h(Y_{1i}|S_{1i},X_{2i}) - h(Z_{1i}) + h(Y_{2i}) - h(Z_{2i})] \end{aligned}$$

where (a) follows from the fact that adding side information increases mutual information; (b) follows from $h(Y_1^N,S_1^N|W_1,W_2) = \sum[h(Z_{1i})+h(Z_{2i})]$ (see Claim 3); (c) follows from the fact that $h(S_1^N|W_2) = h(Y_2^N|W_2)$ (see Claim 4) and X_2^N is a function of (W_2, S_1^{N-1}) (see Claim 5); (d) follows from the fact that conditioning reduces entropy.

Claim 3: $h(Y_1^N,S_1^N|W_1,W_2) = \sum[h(Z_{1i})+h(Z_{2i})]$.

Proof:

$$\begin{aligned} h(Y_1^N,S_1^N|W_1,W_2) &= \sum h(Y_{1i},S_{1i}|W_1,W_2,Y_1^{i-1},S_1^{i-1}) \\ &\overset{(a)}{=} \sum h(Y_{1i},S_{1i}|W_1,W_2,Y_1^{i-1},S_1^{i-1},X_{1i},X_{2i}) \\ &\overset{(b)}{=} \sum h(Z_{1i},Z_{2i}|W_1,W_2,Y_1^{i-1},S_1^{i-1},X_{1i},X_{2i}) \\ &\overset{(c)}{=} \sum[h(Z_{1i})+h(Z_{2i})], \end{aligned}$$

where (a) follows from the fact that X_{1i} is a function of (W_1, Y_1^{i-1}) and X_{2i} is a function of (W_2, S_1^{i-1}) (by Claim 5); (b) follows from the fact that $Y_{1i} = g_d X_{1i} + g_c X_{2i} + Z_{1i}$ and $S_{1i} = g_c X_{1i} + Z_{2i}$; (c) follows from the memoryless property of the channel and the independence assumption of Z_{1i} and Z_{2i}. ■

Claim 4: $h(S_1^N|W_2) = h(Y_2^N|W_2)$.

Proof:

$$\begin{aligned} h(Y_2^N|W_2) &= \sum h(Y_{2i}|Y_2^{i-1},W_2) \overset{(a)}{=} \sum h(S_{1i}|Y_2^{i-1},W_2) \\ &\overset{(b)}{=} \sum h(S_{1i}|Y_2^{i-1},W_2,X_2^i,S_1^{i-1}) \overset{(c)}{=} h(S_1^N|W_2), \end{aligned}$$

where (a) follows from the fact that Y_{2i} is a function of (X_{2i}, S_{1i}) and X_{2i} is a function of (W_2, Y_2^{i-1}); (b) follows from the fact that X_2^i is a function of (W_2, Y_2^{i-1}) and S_1^{i-1} is a function of (Y_2^{i-1}, X_2^{i-1}); (c) follows from the fact that

Y_2^{i-1} is a function of (X_2^{i-1}, S_1^{i-1}) and X_2^i is a function of (W_2, S_1^{i-1}) (by Claim 5). ■

Claim 5: For all $i \geq 1$, X_1^i is a function of (W_1, S_2^{i-1}) and X_2^i is a function of (W_2, S_1^{i-1}).

Proof: By symmetry, it is enough to prove only one. Notice that X_2^i is a function of $(W_1, W_2, Z_1^{i-2}, Z_2^{i-1})$. In Fig. 2 (a), we can easily see that information of (W_1, Z_1^{i-2}) delivered to the second link must pass through S_1^{i-1}. Also S_1^{i-1} contains Z_2^{i-1}. Therefore, X_2^i is a function of (W_2, S_1^{i-1}). ■

If (R_1, R_2) is achievable, then $\epsilon_N \to 0$ as $N \to \infty$. Hence, we get: $R_1 + R_2 \leq h(Y_1|S_1, X_2) - h(Z_1) + h(Y_2) - h(Z_2)$. Assume that X_1 and X_2 have covariance ρ, i.e., $E[X_1 X_2^*] = \rho$. Then, by straightforward computation, we can easily get the desired result.

■

C. Improvement to the Scheme

Explicit calculation using Theorem 2 and 3 shows that the maximum gap between two bounds is upper-bounded by $\frac{1}{2}\log\left(\frac{32}{3}\right) \approx 1.7075$ bits/s/Hz and occurs at $\alpha \approx 1$ [13]. The worst gap can however be improved. The previous scheme includes the *decode-and-forward* operation at the senders after receiving the feedback. This limits the rate. This rate limitation can be avoided by instead performing *amplify-and-forward*: with feedback, the transmitters get the interference plus noise and then forward it subject to the power constraints. In the first stage, each transmitter k sends codeword X_k^N with rate R_k. In the second stage, with feedback transmitter 1 gets the interference plus noise: $S_2^N = g_c X_2^N + Z_1^{(1),N}$. The complex conjugation technique based on Alamouti's scheme is still applied to make X_1^N and S_2^N well separable. Transmitter 1 and 2 send $\frac{S_2^{N*}}{\sqrt{1+\mathsf{INR}}}$ and $-\frac{S_1^{N*}}{\sqrt{1+\mathsf{INR}}}$, respectively, where $\sqrt{1+\mathsf{INR}}$ a normalization factor. Under Gaussian input distribution, we can compute the rate under MMSE demodulation: $\frac{1}{2}I(X_1; Y_1^{(1)}, Y_1^{(2)})$. Straightforward calculations give

$$R_{\mathsf{sym}} = \frac{1}{2}\log\left(\frac{(1+\mathsf{SNR}+\mathsf{INR})^2 - \frac{\mathsf{SNR}}{1+\mathsf{INR}}}{1+2\mathsf{INR}}\right). \quad (5)$$

With (5) and Theorem 3, we obtain the one-bit-gap result.

Theorem 4: For all channel parameters SNR and INR, we can achieve all rates R up to $\overline{C}_{\mathsf{sym}} - 1$, i.e.,

$$\overline{C}_{\mathsf{sym}} - 1 \leq C_{\mathsf{sym}} \leq \overline{C}_{\mathsf{sym}}. \quad (6)$$

D. Comparison to Related Work [6], [7], [8]

For comparison, we plot the generalized degrees-of-freedom of Kramer's scheme and the outer bounds [7], [8] in Fig. 4. Refer to [13] for detailed derivations. When INR is similar to SNR, his scheme is very close to the outer bound. In fact, it achieves the capacity when $\mathsf{INR} = \mathsf{SNR} - \sqrt{2\mathsf{SNR}}$ [7]. However, if INR is quite different from SNR, it becomes far away from the outer bound.

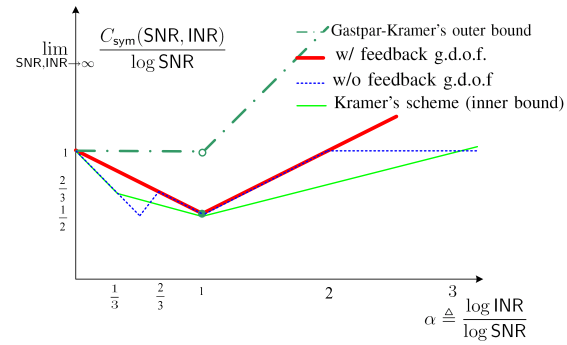

Fig. 4. The generalized degrees-of-freedom of the optimum and Kramer's scheme

V. Conclusion

We found the symmetric capacity to within 1 bit/s/Hz for the two-user Gaussian interference channel with *feedback*. From this result, we discovered a significant role of feedback that it could provide *unbounded* gain in *many-to-many* channels. Interestingly, unbounded gain could be obtained even in the weak interference regime where feedback provides no better alternative path. The gain comes from using feedback to fully exploit the *broadcast* nature of the wireless medium.

References

[1] C. E. Shannon, "The zero error capacity of a noisy channel," *IRE Transactions on Information Theory*, Sept. 1956.

[2] N. T. Gaarder and J. K. Wolf, "The capacity region of a multiple-access discrete memoryless channenl can increase with feedback," *IEEE Transactions on Information Theory*, Jan. 1975.

[3] L. H. Ozarow, "The capacity of the white Gaussian multiple access channel with feedback," *IEEE Transactions on Information Theory*, July 1984.

[4] S. Avestimehr, S. Diggavi, and D. Tse, "A deterministic approach to wireless relay networks," *Proceedings of Allerton Conference on Communication, Control, and computing.*, Sept. 2007.

[5] R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Transactions on Information Theory*, vol. 54, pp. 5534–5562, Dec. 2008.

[6] G. Kramer, "Feedback strategies for white Gaussian interference networks," *IEEE Transactions on Information Theory*, vol. 48, pp. 1423–1438, June 2002.

[7] G. Kramer, "Correction to "Feedback strategies for white Gaussian interference networks", and a capacity theorem for Gaussian interference channels with feedback," *IEEE Transactions on Information Theory*, vol. 50, June 2004.

[8] M. Gastpar and G. Kramer, "On noisy feedback for interference channels," *In Proc. Asilomar Conference on Signals, Systems, and Computers*, Oct. 2006.

[9] J. Jiang, Y. Xin, and H. K. Garg, "Discrete memoryless interference channels with feedback," *CISS 41st Annual Conference*, pp. 581–584, Mar. 2007.

[10] G. Bresler and D. Tse, "The two-user Gaussian interference channel: a deterministic view," *European Transactions on Telecommunications*, June 2008.

[11] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Select Areas in Communications*, vol. 16, pp. 1451–1458, Oct. 1998.

[12] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Transactions on Information Theory*, vol. 49, pp. 2415–2425, Oct. 2003.

[13] C. Suh and D. Tse, "Feedback capacity of the Gaussian interference channel to within 1.7075 bits: the symmetric case," *arXiv:0901.3580v1*, Jan. 2009.