Fiftieth Annual Allerton Conference
Allerton House, UIUC, Illinois, USA
October 1 - 5, 2012

Feedback Increases the Degrees of Freedom of Two Unicast Gaussian Networks

I-Hsiang Wang
EPFL, Lausanne, Switzerland
i-hsiang.wang@epfl.ch

Changho Suh
KAIST, Daejeon, South Korea
chsuh@kaist.ac.kr

***Abstract*—In this paper we investigate the degrees of freedom of layered two unicast Gaussian networks with destination-to-source feedback. The feedback model allows the destinations to feedback their received signals to their respective sources perfectly. Based upon the non-feedback degrees-of-freedom result by Shomorony and Avestimehr [1], we show that in general, feedback enlarges the degrees of freedom region for two unicast layered Gaussian networks. This is the first case in literature where feedback can provide a *degrees-of-freedom* gain in multi-user interference networks.**

I. Introduction

The role of feedback in multi-user networks has received great attention in recent years. For Gaussian networks with constant channel coefficients, it is shown in [2] that, in contrast to the case of point-to-point channels, multiple access channels, and broadcast channels, feedback can provide an *unbounded* capacity gain in two-user interference channels. This is the first Gaussian channel reported in literature where feedback can provide unbounded capacity gain. Such a gain can be viewed as a *generalized degrees of freedom* [3] gain.

Channel output feedback in single-hop interference channels with constant channel coefficients has been studied for two-user interference channels [4] [2] [5] and K-user interference channels [6]. In single-hop interference networks, feedback provides an efficient way to exploit interference received at destinations as useful *side information*. Notably, among the known results in literature, however, feedback does not provide gain in the *degrees of freedom* compared to the non-feedback case for generic channel coefficients[1].

With a good understanding on the role of feedback in single-hop multi-user networks, one of the natural follow-up questions is whether or not this understanding extends to networks with an arbitrary number of nodes and arbitrary connectivity. In particular, can feedback provide a gain in degrees of freedom in general multi-hop networks? In this paper we make a step towards answering the former question by studying the layered two unicast Gaussian networks with single-antenna full-duplex terminals. Furthermore, we answer the latter question in the positive, that feedback can increase the degrees of freedom in two unicast Gaussian networks. This class of networks was studied in [1] when feedback is not available. The non-feedback degrees of freedom (DoF) region is completely characterized in [1]. As a result, the class of layered two unicast Gaussian networks can be partitioned into five different categories according to their DoF regions: $\{\mathfrak{T}, \mathfrak{T}_{12}, \mathfrak{T}_{21}, \mathfrak{P}, \mathfrak{S}\}$, as shown in Fig. 1.

Our main result is that, when feedback is available from the destinations to their respective sources, it helps increase the DoF region to the pentagon region $\mathfrak{P}$ for networks with the non-feedback DoF region $\mathfrak{T}_{12}$ or $\mathfrak{T}_{21}$. To the best of our knowledge, this is the first instance where feedback can increase the degrees of freedom. However, the DoF region cannot go beyond to the pentagon region $\mathfrak{P}$, and hence there is no increase in the sum DoF. On the other hand, compared to the characterization of the non-feedback DoF region where bounds on $2d_1 + d_2$ or $d_1 + 2d_2$ are active ((d_1, d_2) is the achievable DoF pair), here these bounds are no longer active in characterizing the feedback DoF region. Therefore feedback helps balance the resource utilization among different users, and the role of feedback is similar to that in the two-user interference channel [2].

For the achievability, we provide a coding scheme that exploits feedback for utilizing side information at destinations for networks with non-feedback DoF region $\mathfrak{T}_{12}$ (or $\mathfrak{T}_{21}$), so that the DoF pair $(1, 1/2)$ (or $(1/2, 1)$) is achievable. Interestingly, using only the feedback from destination 2 to source 2 suffices to achieve $(1, 1/2)$, and symmetrically, the feedback from destination 1 to source 1 solely suffices to achieve $(1/2, 1)$. The scheme is essentially the same as that in a class of linear deterministic networks considered in the previous work [7]. For the outer bounds, we generalize the proof of the linear deterministic outer bounds [7] by introducing new functional relations and genie-aided techniques to deal with additive Gaussian noise terms, and show that the sum DoF outer bounds still hold with the presence of feedback. The converse proof for the Gaussian case is the main contribution of this paper.

The rest of the paper is organized as follows. In Section II, we formulate the problem and define the network model. In Section III, we introduce an example network on which we focus throughout this conference paper and present our main result: feedback can enlarge its DoF region from $\mathfrak{T}_{12}$ to $\mathfrak{P}$. The achievability proof is presented in Section IV, and the converse proof is detailed in Section V. Finally Section VI concludes the paper with a few remarks on general layered two unicast Gaussian networks.

[1] [6] provided examples of three-user interference channels with specific *integer* channel coefficients where feedback can increase the degrees of freedom, but not for generic channel coefficients

978-1-4673-4539-2/12/$31.00 ©2012 IEEE

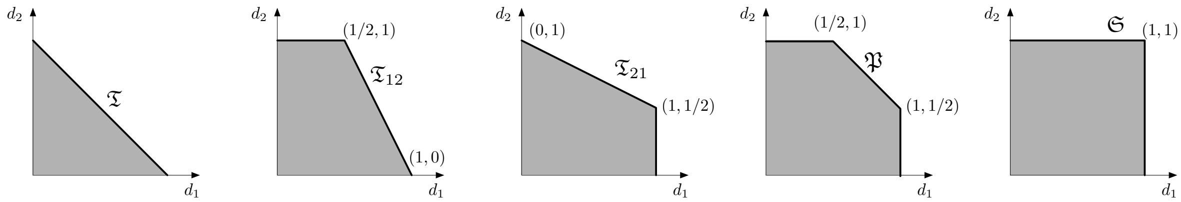


Fig. 1. DoF Regions for Two Unicast Gaussian Network. Without Feedback: $\{\mathfrak{T},\mathfrak{T}_{12},\mathfrak{T}_{21},\mathfrak{P},\mathfrak{S}\}$ [1]. With Feedback:$\{\mathfrak{T},\mathfrak{P},\mathfrak{S}\}$.

II. Problem Formulation

A two-source-two-destination layered network $\mathcal{G}=(\mathcal{V},\mathcal{E})$ is a collection of nodes $\mathcal{V}$ that can be partitioned into $L+2$ *layers* ($L\geq 0$):

$$\mathcal{V}=\bigcup_{k=0}^{L+1}\mathcal{L}_k,\quad \mathcal{L}_k\cap\mathcal{L}_j\neq\emptyset,\ \forall k\neq j,$$

such that for any edge $(\mathsf{u},\mathsf{v})\in\mathcal{E}$, $\exists\ k,\ 0\leq k\leq L$ s.t. $\mathsf{u}\in\mathcal{L}_k,\mathsf{v}\in\mathcal{L}_{k+1}$. The first layer $\mathcal{L}_0=\{\mathsf{s}_1,\mathsf{s}_2\}$ consists of the two source nodes, and the last layer $\mathcal{L}_{L+1}=\{\mathsf{d}_1,\mathsf{d}_2\}$ consists of the two destination nodes. Without loss of generality we assume each node in the network can be reached by at least one of the source nodes and can reach at least one of the destination nodes. For each node $\mathsf{v}\in\mathcal{V}\setminus\{\mathsf{s}_1,\mathsf{s}_2\}$, we define nodes that can reach v as its *predecessors*. Let $\mathcal{P}(\mathsf{v})$ denote the set of predecessors that can reach v in one step. We will call the nodes in $\mathcal{P}(\mathsf{v})$ as the *parents* of v.

Each source s_i, $i=1,2$, has its own message W_i to be transmitted to its own destination d_i at rate R_i, and $\{W_1,W_2\}$ are independent. Let $X_{\mathsf{u}}[t]\in\mathbb{R}$ and $Y_{\mathsf{u}}[t]\in\mathbb{R}$ denote the transmitted and received signals of node u at time t respectively. Due to the causal processing at each node, for any $t\geq 1$,

- $X_{\mathsf{s}_i}[t]$ is a function of $\left(W_i,Y_{\mathsf{d}_i}^{(t-1)}\right)$, for $i=1,2$
- $X_{\mathsf{u}}[t]$ is a function of $Y_{\mathsf{u}}^{(t-1)}$, for $\mathsf{u}\in\mathcal{V}\setminus\{\mathsf{s}_1,\mathsf{s}_2\}$.

The notation $X^t:=\{X[1],\ldots,X[t]\}$ for any $t\geq 1$ and $X^0:=\emptyset$.

Note that in the above formulation, we do not include the feedback links from the destinations to their respective sources into the graph $\mathcal{G}$. Hence, throughout this paper, any graph theoretic properties and conditions are associated to the graph $\mathcal{G}$ itself, not including the feedback links.

For each edge $(\mathsf{u},\mathsf{v})\in\mathcal{E}$, the associated channel coefficient is given by $h_{\mathsf{vu}}\in\mathbb{R}$, where the channel coefficients are drawn from a continuous distribution, i.i.d. over all edges $\mathcal{E}$. Recall that $X_{\mathsf{u}}[t]$ and $Y_{\mathsf{u}}[t]$ denote the transmission and reception of node u respectively. The reception of a node is the superposition of the transmission of its parents along with an additive white Gaussian noise:

$$Y_{\mathsf{v}}[t]=\sum_{\mathsf{u}\in\mathcal{P}(\mathsf{v})}h_{\mathsf{vu}}X_{\mathsf{u}}[t]+Z_{\mathsf{u}}[t],$$

where the noise $Z_{\mathsf{u}}[t]\sim\mathcal{N}(0,\sigma^2)$ is i.i.d. over time and $\mathcal{V}$. Each node has the same power constraint P.

We are mainly interested in the high-SNR approximate capacity region, namely, the collection of achievable degrees of freedom (DoF). For user $i,i=1,2$, its achievable DoF is defined as

$$d_i:=\lim_{P\to\infty}\frac{R_i}{\frac{1}{2}\log\left(1+\frac{P}{\sigma^2}\right)},$$

where (R_1,R_2) lies in the capacity region of the two unicast network.

III. Main Result

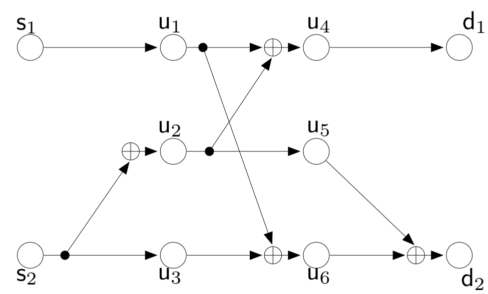


Fig. 2. A Network with Non-Feedback DoF $\mathfrak{T}_{12}$ and Feedback DoF $\mathfrak{P}$

In this conference paper, we shall focus on an example network depicted in Fig. 2 for simplicity of presentation. It is not hard to verify that the non-feedback DoF region of this network is $\mathfrak{T}_{12}$, which can be done by either using the result in [1] or directly proving it (we will re-derive the non-feedback result for this network later in Section IV and Section V). Our main result pertains to the degrees of freedom region when destination-to-source feedback is available.

Theorem 3.1 (Feedback Increases Degrees of Freedom): With destination-to-source feedback, the degrees of freedom region is enlarged from $\mathfrak{T}_{12}$ to $\mathfrak{P}$, where

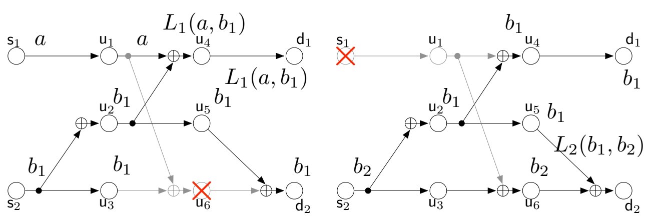
$$\mathfrak{T}_{12}:=\{(d_1,d_2):d_1\geq 0,0\leq d_2\leq 1,2d_1+d_2\leq 2\}$$
$$\mathfrak{P}:=\{(d_1,d_2):0\leq d_1\leq 1,0\leq d_2\leq 1,2d_1+2d_2\leq 3\}.$$

Moreover, using only the feedback from d_2 to s_2 suffices to achieve the feedback DoF region $\mathfrak{P}$.

IV. Achievability

We first argue that without feedback, degrees of freedom pairs (d_1,d_2) that lie in the region $\mathfrak{T}_{12}$ can be achieved. We then show that with feedback, DoF $(1,1/2)$ can be achieved and hence so can the region $\mathfrak{P}$.

It is trivial to see that DoF $(1,0)$ and $(0,1)$ are achievable since s_1 can reach d_1 and s_2 can reach d_2. To achieve DoF $(1/2,1)$, we would like to deliver one real symbol $\{a\}$ from s_1 to d_1 and two real symbols $\{b_1,b_2\}$ from s_2 to d_2 over two time slots. See Fig. 3 for an illustration. At the first time slot, each node transmits what it receives *except* the node u_6, with s_1 transmitting a and s_2 transmitting b_1. The node u_6 keeps silent at the first time slot. Hence d_1 receives a linear combination of $\{a,b_1\}$, while d_2 receives b_1. At the second time slot, each node simply forwards its received signal *except* the node u_2, with s_1 transmitting nothing and s_2 transmitting b_2. The node u_2 transmits its received signal at the first time slot (a scaled version of b_1). Hence d_1 receives a scaled version of b_1, while d_2 receives a linear combination of $\{b_1,b_2\}$. Destination d_1 can now decode a by first decoding interference b_1 from the reception of the second time slot and then canceling b_1 from the reception of the first time slot. Destination d_2 can decode b_1 from the reception of the first time slot and hence b_2 from the reception of the second time slot. DoF $(d_1,d_2)=(1/2,1)$ is achieved.

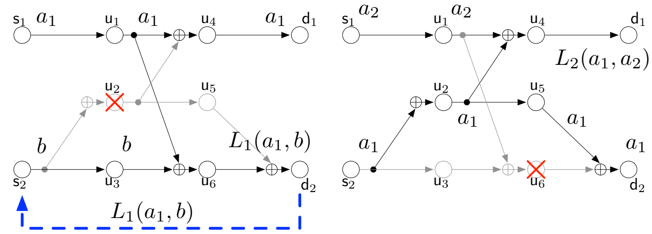


(a) Time Slot 1 (b) Time Slot 2

Fig. 3. A Scheme Achieving $(1/2,1)$ without feedback. $L_1(a,b_1)$ and $L_2(b_1,b_2)$ denote linear combinations of $\{a,b_1\}$ and $\{b_1,b_2\}$ respectively.

When feedback is available, the scheme is similar to that proposed in the linear deterministic two unicast networks with feedback [7]. See Fig. 4 for an illustration. We aim to deliver two real symbols $\{a_1,a_2\}$ from s_1 to d_1 and one real symbol $\{b\}$ from s_2 to d_2 over two time slots. At the first time slot, each node transmits what it receives *except* the node u_2, with s_1 transmitting a_1 and s_2 transmitting b. The node u_2 keeps silent at the first time slot. Hence d_1 receives a scaled version of a_1, while d_2 receives a linear combination of $\{a_1,b\}$. At the second time slot, using the feedback from d_2, the source s_2 obtains a_1. At this time slot, each node simply forwards its received signal *except* the node u_6, with s_1 transmitting a_2 and s_2 transmitting a_1. The node u_6 keeps silent at the second time slot. Hence d_1 receives a linear combination of $\{a_1,a_2\}$, while d_2 receives a scaled version of a_1. Destination d_1 can decode a_1 from the reception of the first time slot and hence a_2 from the reception of the second time slot. Destination d_2 can now decode b by first decoding interference a_1 from the reception of the second time slot and then canceling a_1 from the reception of the first time slot. DoF $(d_1,d_2)=(1,1/2)$ is achieved.

From the above discussion, we prove that using only d_2-to-s_2 feedback, one can achieve DoF region $\mathfrak{P}$.



(a) Time Slot 1 (b) Time Slot 2

Fig. 4. A Scheme Achieving $(1,1/2)$ with feedback. $L_2(a_1,b)$ and $L_2(a_1,a_2)$ denote linear combinations of $\{a_1,b\}$ and $\{a_1,a_2\}$ respectively.

V. Converse Proof

The converse part of the theorem is a non-trivial extension of the converse proof in the linear deterministic case [7]. The main difficulty is due to additive Gaussian noise terms in the intermediate nodes, making the Markov relations (that are essential to the proof) complicated.

Below, we provide two key lemmas that immediately prove the DoF outer bound $2d_1+2d_2\leq 3$.

Lemma 5.1: For any achievable rate pairs (R_1,R_2) and block length N,

$$\begin{aligned}&N\left(2R_1+R_2-\epsilon_{1,N}\right)\\&\leq 2N\left(\frac{1}{2}\log P\right)+I\left(W_1;Z^N_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4},Y^N_{\mathsf{u}_2,\mathsf{u}_3}\right)+o\left(\log P\right)\end{aligned}$$

where $\epsilon_{1,N}\to 0$ as $N\to\infty$.

Lemma 5.2: For any achievable rate pairs R_2 and block length N,

$$\begin{aligned}&N\left(R_2-\epsilon_{2,N}\right)+I\left(W_1;Z^N_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4},Y^N_{\mathsf{u}_2,\mathsf{u}_3}\right)\\&\leq N\left(\frac{1}{2}\log P\right)+o\left(\log P\right)\end{aligned}$$

where $\epsilon_{2,N}\to 0$ as $N\to\infty$.

If there were no feedback from destinations to the respective sources, since $Y^N_{\mathsf{u}_2,\mathsf{u}_3}$ is independent of W_1, the term $I\left(W_1;Z^N_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4},Y^N_{\mathsf{u}_2,\mathsf{u}_3}\right)=0$. Using Lemma 5.1, we have for any achievable (R_1,R_2), $\lim_{P\to\infty}\frac{2R_1+R_2}{\frac{1}{2}\log P}\leq 2$ and hence the DoF outer bound $2d_1+d_2\leq 2$. This recovers the non-feedback DoF result [1].

On the other hand, if feedback is available, in general the term $I\left(W_1;Z^N_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4},Y^N_{\mathsf{u}_2,\mathsf{u}_3}\right)>0$. Instead, we further bound R_2 using Lemma 5.2 and combine it with the bound on $2R_1+R_2$ in Lemma 5.1 to show that for any achievable (R_1,R_2), $\lim_{P\to\infty}\frac{2R_1+2R_2}{\frac{1}{2}\log P}\leq 3$ and hence $2d_1+2d_2\leq 3$. This completes the converse proof.

The high-level idea of the proof of these lemmas goes as follows. One key to the proof, as mentioned in the deterministic case [7], is to deal with the input and output signals at a few "critical" nodes that are in the middle layers of the network rather than those at the source and destination nodes which may be far apart. In the deterministic case, this can be done by using the functional relations induced by the structure of the graph. In the Gaussian case, however, we will have to deal with the leftover additive Gaussian noise. Therefore in subsequent derivations, we first use genie-aided techniques to provide *side information* from which the transmitted signals from the parents of d_1 (or d_2) can be generated. After removing the transmitted signals from the parents, only the additive noise $Z_{\mathsf{d}_1}^N$ (or $Z_{\mathsf{d}_2}^N$), which provides zero DoF, is left. Next we let the genie provide more side information to facilitate recombination and cancellation of terms. Finally, we evaluate the DoF upper bound for each differential entropy terms to complete the proof.

The rest of this section is devoted to the proof of Lemma 5.1 and Lemma 5.2.

A. *Proof of Lemma 5.1*

We begin with a few notations to facilitate the discussion.

- For a link $(\mathsf{u},\mathsf{v}) \in \mathcal{E}$, define $X_{\mathsf{u}|\mathsf{v}} := h_{\mathsf{vu}} X_{\mathsf{u}} + Z_{\mathsf{v}}$.
- For notational convenience, for a set of nodes $\mathcal{A}$, $X_{\mathcal{A}}$ denotes the collection of random variables $\{X_{\mathsf{a}} : \mathsf{a} \in \mathcal{A}\}$. In the context of no confusion, in the subscript we drop the big parentheses of $\mathcal{A}$.
- $A \overset{\mathrm{f}}{=} B$ denotes "A is a function of B"

By Fano's inequality, for any achievable rate pair (R_1, R_2),

$$\begin{aligned} &N(2R_1 + R_2 - \epsilon_{1,N}) \\ &\le I\left(W_1; Y_{\mathsf{d}_1}^N\right) + I\left(W_2; Y_{\mathsf{d}_2}^N\right) + I\left(W_1; Y_{\mathsf{d}_1}^N\right), \end{aligned}$$

where $\epsilon_{1,N} \to 0$ as $N \to \infty$.

As mentioned before, we would like to use genie-aided techniques to remove the transmitted signals from the parents of d_1 and d_2. Using data processing inequality, we upper bound the three terms as follows:

$$\begin{aligned} I\left(W_1; Y_{\mathsf{d}_1}^N\right) &\le I\left(W_1; Y_{\mathsf{d}_1,\mathsf{u}_4}^N\right) \\ &\le I\left(W_1; Z_{\mathsf{d}_1}^N, Y_{\mathsf{u}_4}^N\right), \\ I\left(W_2; Y_{\mathsf{d}_2}^N\right) &\le I\left(W_2; Y_{\mathsf{d}_2,\mathsf{u}_5,\mathsf{u}_6}^N\right) \\ &\le I\left(W_2; Z_{\mathsf{d}_2}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N\right), \\ I\left(W_1; Y_{\mathsf{d}_1}^N\right) &\le I\left(W_1; Y_{\mathsf{d}_1,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_3|\mathsf{u}_6}^N\right) \\ &\le I\left(W_1; Z_{\mathsf{d}_1}^N, Y_{\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_3|\mathsf{u}_6}^N\right). \end{aligned}$$

The above upper bounds for the three terms are due to the following functional relations:

$$\begin{aligned} &X_{\mathsf{u}_4}^N \overset{\mathrm{f}}{=} Y_{\mathsf{u}_4}^N \overset{\mathrm{f}}{=} \left(X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_1}^N\right) \overset{\mathrm{f}}{=} \left(X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_3|\mathsf{u}_6}^N, Y_{\mathsf{u}_6}^N\right), \\ &X_{\mathsf{u}_5}^N \overset{\mathrm{f}}{=} Y_{\mathsf{u}_5}^N, \ X_{\mathsf{u}_6}^N \overset{\mathrm{f}}{=} Y_{\mathsf{u}_6}^N. \end{aligned}$$

Next, we provide more genie-aided side information to facilitate recombination and cancellation of terms to proceed further.

$$\begin{aligned} &N(2R_1 + R_2 - \epsilon_{1,N}) \\ &\le I\left(W_1; Z_{\mathsf{d}_1}^N, Y_{\mathsf{u}_4}^N\right) + I\left(W_2; Z_{\mathsf{d}_2}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N\right) \\ &\quad + I\left(W_1; Z_{\mathsf{d}_1}^N, Y_{\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_3|\mathsf{u}_6}^N\right) \\ &\le I\left(W_1; Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N\right) \\ &\quad + I\left(W_2; Z_{\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3,\mathsf{d}_1}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N\right) \\ &\quad + I\left(W_1; Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_3|\mathsf{u}_6}^N, W_2\right) \\ &\overset{\text{(a)}}{=} I\left(W_1; Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N\right) \\ &\quad + I\left(W_2; Z_{\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3,\mathsf{d}_1}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N\right) \\ &\quad + I\left(W_1; Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_3|\mathsf{u}_6}^N \middle| W_2\right) \\ &= h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N\right) - h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N \middle| W_1\right) \\ &\quad + h\left(Z_{\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3,\mathsf{d}_1}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N\right) \\ &\quad - h\left(Z_{\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3,\mathsf{d}_1}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N \middle| W_2\right) \\ &\quad + h\left(Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_3|\mathsf{u}_6}^N \middle| W_2\right) \\ &\quad - h\left(Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_3|\mathsf{u}_6}^N \middle| W_1, W_2\right) \\ &= h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N\right) + h\left(Z_{\mathsf{d}_2,\mathsf{u}_3}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N \middle| Z_{\mathsf{d}_1,\mathsf{u}_1}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N\right) \\ &+ \left[-h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N \middle| W_1\right) + h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N\right)\right] \\ &\quad + \left[\begin{array}{l} -h\left(Z_{\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3,\mathsf{d}_1}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N \middle| W_2\right) \\ +h\left(Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_3|\mathsf{u}_6}^N \middle| W_2\right) \end{array}\right] \\ &\quad - h\left(Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_3|\mathsf{u}_6}^N \middle| W_1, W_2\right) \\ &= h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N\right) + h\left(Z_{\mathsf{d}_2,\mathsf{u}_3}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N \middle| Z_{\mathsf{d}_1,\mathsf{u}_1}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N\right) \\ &\quad + \left[-h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N \middle| W_1\right) + h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N\right)\right] \\ &\quad + h\left(X_{\mathsf{u}_3|\mathsf{u}_6}^N \middle| W_2, Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N\right) \\ &\quad - h\left(Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}^N, Y_{\mathsf{u}_5,\mathsf{u}_6}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N, X_{\mathsf{u}_3|\mathsf{u}_6}^N \middle| W_1, W_2\right). \end{aligned}$$

Here (a) holds since $\{W_1, W_2\}$ are independent.

To this end, we observe that to combine the terms $\left[-h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N \middle| W_1\right) + h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N\right)\right]$, it is desired to have $h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N \middle| W_1\right) = h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N \middle| W_1\right)$. Hence, we manipulate the term $h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N \middle| W_1\right)$ as follows to proceed:

$$\begin{aligned} &h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, Y_{\mathsf{u}_4}^N \middle| W_1\right) \\ &= \sum_{t=1}^N h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}[t], Y_{\mathsf{u}_4}[t] \middle| W_1, Z_{\mathsf{d}_1,\mathsf{u}_1}^{(t-1)}, Y_{\mathsf{u}_4}^{(t-1)}\right) \\ &\overset{\text{(b)}}{=} \sum_{t=1}^N h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}[t], X_{\mathsf{u}_2|\mathsf{u}_4}[t] \middle| W_1, Z_{\mathsf{d}_1,\mathsf{u}_1}^{(t-1)}, X_{\mathsf{u}_2|\mathsf{u}_4}^{(t-1)}\right) \\ &= h\left(Z_{\mathsf{d}_1,\mathsf{u}_1}^N, X_{\mathsf{u}_2|\mathsf{u}_4}^N \middle| W_1\right). \end{aligned}$$

Here (b) is due to the fact that

$$\begin{aligned} X_{\mathsf{u}_1}^t &\overset{\mathrm{f}}{=} \left(X_{\mathsf{s}_1}^{(t-1)}, Z_{\mathsf{u}_1}^{(t-1)}\right) \overset{\mathrm{f}}{=} \left(W_1, Y_{\mathsf{d}_1}^{(t-2)}, Z_{\mathsf{u}_1}^{(t-1)}\right) \\ &\overset{\mathrm{f}}{=} \left(W_1, Y_{\mathsf{u}_4}^{(t-3)}, Z_{\mathsf{d}_1}^{(t-2)}, Z_{\mathsf{u}_1}^{(t-1)}\right). \end{aligned}$$

Therefore, we have

$$
\begin{aligned}
&N(2R_1 + R_2 - \epsilon_{1,N}) \\
&\le h\left(Z^N_{\mathsf{d}_1,\mathsf{u}_1}, Y^N_{\mathsf{u}_4}\right) + h\left(Z^N_{\mathsf{d}_2,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}\middle| Z^N_{\mathsf{d}_1,\mathsf{u}_1}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right) \\
&\quad + \left[-h\left(Z^N_{\mathsf{d}_1,\mathsf{u}_1}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\middle| W_1\right) + h\left(Z^N_{\mathsf{d}_1,\mathsf{u}_1}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right)\right] \\
&\quad + h\left(X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_2, Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right) \\
&\quad - h\left(Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}, X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_1, W_2\right) \\
&= h\left(Z^N_{\mathsf{d}_1,\mathsf{u}_1}, Y^N_{\mathsf{u}_4}\right) + h\left(Z^N_{\mathsf{d}_2,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}\middle| Z^N_{\mathsf{d}_1,\mathsf{u}_1}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right) \\
&\quad + I\left(W_1; Z^N_{\mathsf{d}_1,\mathsf{u}_1}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right) \\
&\quad + h\left(X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_2, Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right) \\
&\quad - h\left(Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}, X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_1, W_2\right) \\
&\overset{(c)}{\le} h\left(Z^N_{\mathsf{d}_1,\mathsf{u}_1}, Y^N_{\mathsf{u}_4}\right) + h\left(Z^N_{\mathsf{d}_2,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}\middle| Z^N_{\mathsf{d}_1,\mathsf{u}_1}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right) \\
&\quad + I\left(W_1; Z^N_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4}, Y^N_{\mathsf{u}_2,\mathsf{u}_3}\right) \\
&\quad + h\left(X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_2, Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right) \\
&\quad - h\left(Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}, X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_1, W_2\right).
\end{aligned}
$$

Here (c) is due to the fact that $X^N_{\mathsf{u}_2|\mathsf{u}_4} \overset{\mathrm{f}}{=} \left(Z^N_{\mathsf{u}_4}, Y^N_{\mathsf{u}_2}\right)$ and that the extra genie-aided side information $Y^N_{\mathsf{u}_3}$ is added.

We prove the following claim regarding the DoF for the differential entropy terms in the above upper bound to complete the proof of Lemma 5.1.

Claim 5.1:

$$\lim_{P\to\infty} \frac{h\left(Z^N_{\mathsf{d}_1,\mathsf{u}_1}, Y^N_{\mathsf{u}_4}\right)}{\frac{1}{2}\log P} \le N$$

$$\lim_{P\to\infty} \frac{h\left(Z^N_{\mathsf{d}_2,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}\middle| Z^N_{\mathsf{d}_1,\mathsf{u}_1}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right)}{\frac{1}{2}\log P} \le N$$

$$\lim_{P\to\infty} \frac{h\left(X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_2, Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right)}{\frac{1}{2}\log P} \le 0$$

$$\lim_{P\to\infty} \frac{-h\left(Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}, X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_1, W_2\right)}{\frac{1}{2}\log P} \le 0$$

Proof:

(1) Since i.i.d. Gaussian input maximizes differential entropy of a random vector,

$$
\begin{aligned}
&h\left(Z^N_{\mathsf{d}_1,\mathsf{u}_1}, Y^N_{\mathsf{u}_4}\right) \le h\left(Z^N_{\mathsf{d}_1,\mathsf{u}_1}\right) + h\left(Y^N_{\mathsf{u}_4}\right) \\
&\le \frac{N}{2}\log\left\{(2\pi e)^2\sigma^4\right\} \\
&\quad + \frac{N}{2}\log\left\{(2\pi e)\left[\left(|h_{\mathsf{u}_4\mathsf{u}_1}| + |h_{\mathsf{u}_4\mathsf{u}_2}|\right)^2 P + \sigma^2\right]\right\}.
\end{aligned}
$$

Hence $\lim_{P\to\infty} \frac{h\left(Z^N_{\mathsf{d}_1,\mathsf{u}_1}, Y^N_{\mathsf{u}_4}\right)}{\frac{1}{2}\log P} \le N$.

(2) We use the intuition that $Y^N_{\mathsf{u}_5}$ and $X^N_{\mathsf{u}_2|\mathsf{u}_4}$ are roughly the same. By the fact the conditioning reduces differential entropy and i.i.d. Gaussian input maximizes conditional differential entropy,

$$
\begin{aligned}
&h\left(Z^N_{\mathsf{d}_2,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}\middle| Z^N_{\mathsf{d}_1,\mathsf{u}_1}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right) \\
&\le h\left(Y^N_{\mathsf{u}_6}\right) + h\left(Z^N_{\mathsf{d}_2,\mathsf{u}_3}\right) + h\left(Y^N_{\mathsf{u}_5}\middle| X^N_{\mathsf{u}_2|\mathsf{u}_4}\right) \\
&\le \frac{N}{2}\log\left\{(2\pi e)\left[\left(|h_{\mathsf{u}_6\mathsf{u}_1}| + |h_{\mathsf{u}_6\mathsf{u}_3}|\right)^2 P + \sigma^2\right]\right\} \\
&\quad + \frac{N}{2}\log\left\{(2\pi e)^2\sigma^4\right\} \\
&\quad + \frac{N}{2}\log\left\{(2\pi e)\frac{\left(|h_{\mathsf{u}_5\mathsf{u}_2}|^2 + |h_{\mathsf{u}_4\mathsf{u}_2}|^2\right)P\sigma^2 + \sigma^4}{|h_{\mathsf{u}_4\mathsf{u}_2}|^2 P + \sigma^2}\right\}.
\end{aligned}
$$

Hence $\lim_{P\to\infty} \frac{h\left(Z^N_{\mathsf{d}_2,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}\middle| Z^N_{\mathsf{d}_1,\mathsf{u}_1}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right)}{\frac{1}{2}\log P} \le N$.

(3) Since

$$X^N_{\mathsf{u}_3} \overset{\mathrm{f}}{=} \left(X^N_{\mathsf{s}_2}, Z^N_{\mathsf{u}_3}\right) \overset{\mathrm{f}}{=} \left(W_2, Y^N_{\mathsf{d}_2}, Z^N_{\mathsf{u}_3}\right) \overset{\mathrm{f}}{=} \left(W_2, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, Z^N_{\mathsf{u}_3,\mathsf{d}_2}\right),$$

we have

$$
\begin{aligned}
&h\left(X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_2, Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right) \\
&\le h\left(X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_2, Z^N_{\mathsf{d}_2,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}\right) \\
&= h\left(X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_2, Z^N_{\mathsf{d}_2,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_3}\right) \\
&= h\left(Z^N_{\mathsf{u}_6}\middle| W_2, Z^N_{\mathsf{d}_2,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_3}\right) \\
&\le h\left(Z^N_{\mathsf{u}_6}\right) = \frac{N}{2}\log(2\pi e)\sigma^2.
\end{aligned}
$$

Hence $\lim_{P\to\infty} \frac{h\left(X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_2, Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}\right)}{\frac{1}{2}\log P} \le 0$.

(4) The last one requires using chain rule to expand the term into a series in time.

$$
\begin{aligned}
&h\left(Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}, X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_1, W_2\right) \\
&= \sum_{t=1}^{N} h\left(\begin{matrix} Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}[t], Y_{\mathsf{u}_5,\mathsf{u}_6}[t], \\ X_{\mathsf{u}_2|\mathsf{u}_4}[t], X_{\mathsf{u}_3|\mathsf{u}_6}[t] \end{matrix}\middle| \begin{matrix} W_1, W_2, Z^{(t-1)}_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, \\ Y^{(t-1)}_{\mathsf{u}_5,\mathsf{u}_6}, X^{(t-1)}_{\mathsf{u}_2|\mathsf{u}_4}, X^{(t-1)}_{\mathsf{u}_3|\mathsf{u}_6} \end{matrix}\right) \\
&\overset{(d)}{\ge} \sum_{t=1}^{N} h\left(\begin{matrix} Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}[t], Y_{\mathsf{u}_5,\mathsf{u}_6}[t], \\ X_{\mathsf{u}_2|\mathsf{u}_4}[t], X_{\mathsf{u}_3|\mathsf{u}_6}[t] \end{matrix}\middle| \begin{matrix} W_1, W_2, Z^{(t-1)}_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, \\ Y^{(t-1)}_{\mathsf{u}_5,\mathsf{u}_6}, X^{(t-1)}_{\mathsf{u}_2|\mathsf{u}_4}, X^{(t-1)}_{\mathsf{u}_3|\mathsf{u}_6}, \\ X_{\mathsf{u}_1,\mathsf{u}_2,\mathsf{u}_3}[t] \end{matrix}\right) \\
&= \sum_{t=1}^{N} h\left(Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3,\mathsf{u}_4,\mathsf{u}_5,\mathsf{u}_6}[t]\middle| \begin{matrix} W_1, W_2, Z^{(t-1)}_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, \\ Y^{(t-1)}_{\mathsf{u}_5,\mathsf{u}_6}, X^{(t-1)}_{\mathsf{u}_2|\mathsf{u}_4}, X^{(t-1)}_{\mathsf{u}_3|\mathsf{u}_6}, \\ X_{\mathsf{u}_1,\mathsf{u}_2,\mathsf{u}_3}[t] \end{matrix}\right) \\
&\overset{(e)}{=} \sum_{t=1}^{N} h\left(Z_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3,\mathsf{u}_4,\mathsf{u}_5,\mathsf{u}_6}[t]\right) = \frac{N}{2}\log(2\pi e)^7\sigma^{14}.
\end{aligned}
$$

Here (d) is due to the fact that conditioning reduces differential entropy, and (e) is due to the fact that the additive noises at time t are independent of the messages (W_1, W_2), any reception up to time $t-1$, and any transmission up to time t. Hence $\lim_{P\to\infty} \frac{-h\left(Z^N_{\mathsf{d}_1,\mathsf{d}_2,\mathsf{u}_1,\mathsf{u}_3}, Y^N_{\mathsf{u}_5,\mathsf{u}_6}, X^N_{\mathsf{u}_2|\mathsf{u}_4}, X^N_{\mathsf{u}_3|\mathsf{u}_6}\middle| W_1, W_2\right)}{\frac{1}{2}\log P} \le 0$. ■

B. Proof of Lemma 5.2

By Fano's inequality and data processing inequality,

$$\begin{aligned} N\left(R_2-\epsilon_{2,N}\right) &\le I\left(W_2; Y_{\mathsf{d}_2}^N\right) \\ &\le I\left(W_2; Y_{\mathsf{d}_2,\mathsf{u}_2,\mathsf{u}_3}^N, Z_{\mathsf{u}_5,\mathsf{u}_6,\mathsf{u}_1,\mathsf{u}_4,\mathsf{d}_1}^N \middle| W_1\right), \end{aligned}$$

where $\epsilon_{2,N} \to 0$ as $N \to \infty$. Combining with the term $I\left(W_1; Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right)$, we have

$$\begin{aligned} & N\left(R_2-\epsilon_{2,N}\right) + I\left(W_1; Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right) \\ &\le I\left(W_2; Y_{\mathsf{d}_2,\mathsf{u}_2,\mathsf{u}_3}^N, Z_{\mathsf{u}_5,\mathsf{u}_6,\mathsf{u}_1,\mathsf{u}_4,\mathsf{d}_1}^N \middle| W_1\right) \\ &\quad + I\left(W_1; Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right) \\ &= h\left(Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right) + h\left(Y_{\mathsf{d}_2}^N \middle| W_1, Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4,\mathsf{u}_5,\mathsf{u}_6}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right) \\ &\quad + h\left(Z_{\mathsf{u}_5,\mathsf{u}_6}^N \middle| W_1, Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right) \\ &\quad - h\left(Y_{\mathsf{d}_2,\mathsf{u}_2,\mathsf{u}_3}^N, Z_{\mathsf{u}_5,\mathsf{u}_6,\mathsf{u}_1,\mathsf{u}_4,\mathsf{d}_1}^N \middle| W_1, W_2\right). \end{aligned}$$

We need to prove a second claim regarding the DoF of the above differential entropy terms to conclude the proof.

Claim 5.2:

$$\begin{aligned} &\lim_{P\to\infty} \frac{h\left(Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right)}{\frac{1}{2}\log P} \le N \\ &\lim_{P\to\infty} \frac{h\left(Y_{\mathsf{d}_2}^N \middle| W_1, Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4,\mathsf{u}_5,\mathsf{u}_6}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right)}{\frac{1}{2}\log P} \le 0 \\ &\lim_{P\to\infty} \frac{h\left(Z_{\mathsf{u}_5,\mathsf{u}_6}^N \middle| W_1, Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right)}{\frac{1}{2}\log P} \le 0 \\ &\lim_{P\to\infty} \frac{-h\left(Y_{\mathsf{d}_2,\mathsf{u}_2,\mathsf{u}_3}^N, Z_{\mathsf{u}_5,\mathsf{u}_6,\mathsf{u}_1,\mathsf{u}_4,\mathsf{d}_1}^N \middle| W_1, W_2\right)}{\frac{1}{2}\log P} \le 0 \end{aligned}$$

Proof: The third and the last item can be proved using techniques similar to that in Claim 5.1 and hence we omit the proof.

(1) Since i.i.d. Gaussian input maximizes differential entropy of a random vector,

$$\begin{aligned} & h\left(Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right) \le h\left(Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4}^N\right) + h\left(Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right) \\ &\le \frac{N}{2}\log\left\{(2\pi e)^3\sigma^6\right\} \\ &\quad + \frac{N}{2}\log\left\{(2\pi e)^2\left[\left(|h_{\mathsf{u}_2\mathsf{s}_2}|^2 + |h_{\mathsf{u}_3\mathsf{s}_2}|^2\right)P\sigma^2 + \sigma^4\right]\right\}. \end{aligned}$$

Hence $\lim_{P\to\infty} \frac{h\left(Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right)}{\frac{1}{2}\log P} \le N$.

(2) First observe the following:

$$\begin{aligned} X_{\mathsf{u}_5,\mathsf{u}_6}^N &\overset{\mathrm{f}}{=} \left(X_{\mathsf{u}_1,\mathsf{u}_2,\mathsf{u}_3}^N, Z_{\mathsf{u}_5,\mathsf{u}_6}^N\right) \overset{\mathrm{f}}{=} \left(X_{\mathsf{u}_1}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N, Z_{\mathsf{u}_5,\mathsf{u}_6}^N\right), \\ X_{\mathsf{u}_1}^N &\overset{\mathrm{f}}{=} \left(Z_{\mathsf{u}_1}^N, W_1, Y_{\mathsf{d}_1}^{(N-1)}\right) \overset{\mathrm{f}}{=} \left(Z_{\mathsf{u}_1,\mathsf{d}_1,\mathsf{u}_4}^N, W_1, X_{\mathsf{u}_1,\mathsf{u}_2}^{(N-1)}\right) \\ &\overset{\mathrm{f}}{=} \left(Z_{\mathsf{u}_1,\mathsf{d}_1,\mathsf{u}_4}^N, W_1, X_{\mathsf{u}_2}^{(N-1)}\right) \overset{\mathrm{f}}{=} \left(Z_{\mathsf{u}_1,\mathsf{d}_1,\mathsf{u}_4}^N, W_1, Y_{\mathsf{u}_2}^N\right). \end{aligned}$$

Hence $X_{\mathsf{u}_5,\mathsf{u}_6}^N \overset{\mathrm{f}}{=} \left(W_1, Y_{\mathsf{u}_2,\mathsf{u}_3}^N, Z_{\mathsf{u}_5,\mathsf{u}_6,\mathsf{u}_1,\mathsf{u}_4,\mathsf{d}_1}^N\right)$, and

$$\begin{aligned} & h\left(Y_{\mathsf{d}_2}^N \middle| W_1, Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4,\mathsf{u}_5,\mathsf{u}_6}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right) \\ &= h\left(Y_{\mathsf{d}_2}^N \middle| W_1, Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4,\mathsf{u}_5,\mathsf{u}_6}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N, X_{\mathsf{u}_5,\mathsf{u}_6}^N\right) \\ &= h\left(Z_{\mathsf{d}_2}^N \middle| W_1, Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4,\mathsf{u}_5,\mathsf{u}_6}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N, X_{\mathsf{u}_5,\mathsf{u}_6}^N\right) \\ &\le h\left(Z_{\mathsf{d}_2}^N\right) = \frac{N}{2}\log(2\pi e)\sigma^2. \end{aligned}$$

Therefore $\lim_{P\to\infty} \frac{h\left(Y_{\mathsf{d}_2}^N \middle| W_1, Z_{\mathsf{d}_1,\mathsf{u}_1,\mathsf{u}_4,\mathsf{u}_5,\mathsf{u}_6}^N, Y_{\mathsf{u}_2,\mathsf{u}_3}^N\right)}{\frac{1}{2}\log P} \le 0$. ■

VI. Concluding Remarks

In this paper we characterize the feedback degrees of freedom region of an example network. The non-feedback degrees of freedom region is $\mathfrak{T}_{12}$, while using a simple coding scheme, the degrees of freedom region $\mathfrak{P}$ can be achieved with feedback. We also develop the matching sum DoF outer bound. In our knowledge, this is the first instance where feedback can increase the degrees of freedom in Gaussian interference networks with generic channel coefficients. However, feedback does not increase the sum DoF.

For general layered two unicast Gaussian networks, we further characterize the feedback degrees of freedom region. It turns out that when feedback is available from the destinations to their respective sources, it helps increase the degrees of freedom region *if and only if* the non-feedback DoF region of the network is $\mathfrak{T}_{12}$ or $\mathfrak{T}_{21}$, and the feedback DoF region is the pentagon region $\mathfrak{P}$. If the non-feedback DoF region is $\mathfrak{T}$, $\mathfrak{P}$, or $\mathfrak{S}$, feedback does not help increase the degrees of freedom. The complete characterization will appear in an extended version of this conference paper.

Acknowledgement

The work of I-Hsiang Wang was supported by EU project CONECT FP7-ICT-2009-257616.

References

[1] I. Shomorony and A. S. Avestimehr, "Two-unicast wireless networks: Characterizing the degrees-of-freedom," *Submitted to IEEE Transactions on Information Theory*, March 2011, http://arxiv.org/abs/1102.2498.

[2] C. Suh and D. N. C. Tse, "Feedback capacity of the Gaussian interference channel to within 2 bits," *IEEE Transactions on Information Theory*, vol. 57, pp. 2667–2685, May 2011.

[3] R. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Transactions on Information Theory*, vol. 54, pp. 5534–5562, December 2008.

[4] G. Kramer, "Feedback strategies for white Gaussian interference networks," *IEEE Transactions on Information Theory*, vol. 48, pp. 1423–1438, June 2002.

[5] C. Suh, "Degrees-of-freedom region of the two-user Gaussian MIMO interference channel with feedback," *Unpublished Note*, July 2012, http://sites.google.com/site/changhosuh/fdof_MIMOIC.pdf.

[6] D. S. Papailiopoulos, C. Suh, and A. G. Dimakis, "Feedback in the K-user interference channel," *Proceedings of IEEE International Symposium on Information Theory*, pp. 3140–3144, July 2012.

[7] I.-H. Wang, "On two unicast wireless networks with destination-to-source feedback," *Proceedings of IEEE International Symposium on Information Theory*, pp. 41–45, July 2012.