Efficient Algorithm for Proportional Fairness Scheduling in Multicast OFDM Systems

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***Abstract*— In multicast orthogonal frequency division multiplexing (OFDM) systems, the difference in link conditions of users complicates adaptive modulation because modulation should be adjusted to serve the user who experiences the worst channel condition. If we assume that the multicast data are separated into layers and any combination of the layers can be decoded at the receiver, the network throughput can be increased by performing subcarrier/bit allocation [1]. In addition, if we consider the concept of proportional fairness (PF), the fairness factor can be increased while minimizing total throughput degradation. In this paper, we formulate the optimization problem for PF scheduling and show that this problem is NP-hard one requiring large complexity. To reduce the complexity, we propose a simple heuristic algorithm for PF scheduling by separating subcarrier allocation and bit loading. Numerical results show that the performance difference between the optimum and proposed algorithms is within about 5%, and that PF scheduling may be the best solution for multicast scheduling if we consider both total throughput and fairness.**

I. Introduction

Multicasting delivers data to a group of users by a single transmission, which is particularly useful for high-data-rate multimedia service due to its ability to save the network resources. Since the bandwidth allocated to each user is different in heterogeneous network, the data rate of multicast stream is limited by the data rate of the least capable user, otherwise it is not delivered to several users. One approach of solving the heterogeneity is to exploit hierarchy in multicast data [2], [3], [4], [5]. For example, raw video data is compressed into a number of layers, arranged in a hierarchy that provides progressive refinement. If only the first layer is received by the user with the lowest data rate, the decoder produces the worst quality version. As more layers are received by more capable users, the decoder combines the layers to produce improved quality.

In wireless system, the spectrum is very scarce and the channel varies according to users due to Rayleigh fading; therefore, the multicasting in wireless network should be spectrally efficient and be able to cope with the channel variation. The channel variation among users complicates adaptive modulation because the modulation should be adjusted to serve the user who experiences the worst channel condition; thus, it is usually adapted to the worst link condition [6], or the heterogeneity in link condition is often ignored during the design of adaptive modulation for multicast transmission [7]. To cope with the channel variation between users without adaptation, the non-uniform phase-shift-keying (PSK) is used in [4] where the base layer data is encoded to constellation points that are far apart in distance from each other than the higher layer data are encoded to. In [8], an adaptive modulation for multicast data is proposed assuming that the same modulation is used for all the subcarriers in an OFDM symbol. In [1], a dynamic subcarrier/bit allocation method was proposed for multicast OFDM system in a way that maximizes the total data rate of all the users. In practical systems, however, the fairness factor should be considered altogether with total throughput. Proportional fairness (PF) algorithm can be suitable for improving fairness without a large degradation of total throughput.

In this paper, we formulate the optimization problem for PF scheduling in multicast OFDM systems, and show that this problem is NP-hard one requiring large complexity. To reduce the complexity, we propose a simple heuristic algorithm by separating subcarrier allocation and bit loading: firstly, with the initial value for maximizing total throughput, subcarrier allocation is performed in a way that maximizes the product of data rates of all users; in the second step, the number of bits loaded to each subcarrier is determined using the modified Levin-Campello algorithm, which has been already developed in [1]. Numerical results show that the performance difference between the optimum and proposed algorithms is within about 5%, and that PF scheduling may be the best solution for multicast scheduling if we consider both total throughput and fairness with the same importance.

II. Problem Formulation

Multicast OFDM transmitter and receiver supporting K users are shown in Fig. 1. It is assumed that the multicast data are separated into layers, and any combination of the layers can be decoded at the receiver. The multicast data are fed into the adaptive modulator that assigns each subcarrier to a group of users who receive the same multicast data, and determines the number of bits on each subcarrier considering the lowest one among the channel gains of all users allocated to that subcarrier; therefore, channel information about all subcarriers of all users should be known to the transmitter, and

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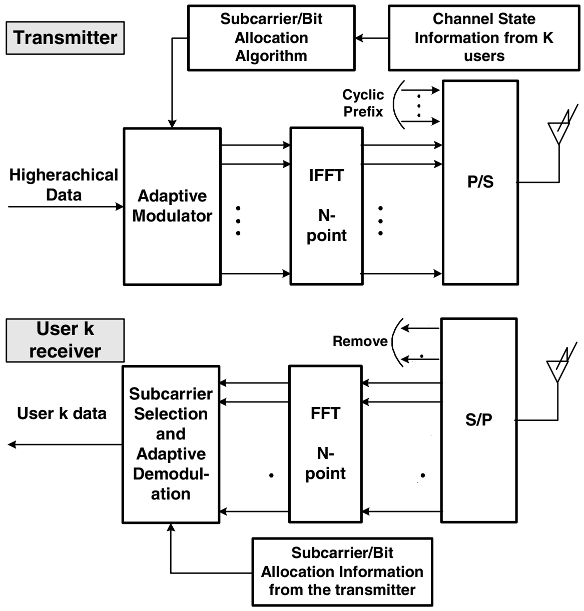

Fig. 1. Multicast OFDM Systems with Hierarchical Data

the subcarrier/bit allocation information should be transmitted to each user through a separate control channel. Since the subcarrier/bit allocation information is available at the k-th user, subcarriers allocated to the user are selected and the signals associated with the subcarriers are demodulated, and then the signals are combined to reconstruct the original multicast data.

If we assume that the perceived quality of multicast data is proportional to the amount of data received by each user, the adaptive modulator should allocate subcarrier and load bits in a way that maximizes the total number of bits received by all the users. For this, the optimum and suboptimum algorithms has been already provided in [1]. In this paper, we focus on the specific algorithms for PF scheduling considering both fairness and total throughput. To describe the optimization procedure, we introduce notations that are adopted in [1]. Let R_k be the data rate of the k-th user and c_n be the number of bits that are assigned to the n-th subcarrier. Here, the user index k is unnecessary because the users in the group receive the identical data using the same modulation. It is assumed that $c_n \in \mathbf{D} = \{0, 1, \cdots, M\}$ where M is the maximum number of bits/symbol that can be transmitted by each subcarrier. The data rate R_k can be expressed as $R_k = \sum_{n=1}^{N} c_n \rho_{k,n}$, where $\rho_{k,n}$ is a binary value indicating whether the k-th user utilizes the n-th subcarrier or not.

$$\rho_{k,n} = \begin{cases} 1, & \text{if } n\text{-th subcarrier is used for } k\text{-th user} \\ 0. & \text{else} \end{cases} \tag{1}$$

The transmission power allocated to the n-th subcarrier is

$$P_n = \max_k P_{k,n} = \max_k \left(\frac{f(c_n)\rho_{k,n}}{\alpha_{k,n}^2} \right) \tag{2}$$

where $f(c_n)$ is the required receive power in the n-th subcarrier for reliable reception of c_n when the channel gain is unity. In practical system, if channel coding is considered in addition to adaptive modulation, $f(c_n)$ should be simply replaced by $g(c_n, r_n)$ that can be numerically or analytically calculated for code rate r_n. The parameter, $\alpha_{k,n}^2$, indicates the channel gain of the n-th subcarrier of the k-th user. Since subcarrier can be shared by more than one user, maximum transmit power should be selected among the required transmit powers of selected users.

In multicast systems with hierarchical data, data rate depends highly on channel quality; hence, it is meaningful to solve the rate adaptive (RA) problem having the power constraint. The fundamental problem of multicast system is that total throughput is reduced due to the dependency on the lowest channel gain. Thus, it is important to solve the optimization problem for maximizing total data rate of all the users. However, in practical systems, we should consider not only total throughput but also fairness. If we consider the fairness between users, the PF scheduling is suitable because it tries to improve the fairness altogether with increasing the network throughput. Although max-min fairness scheduling guarantees the fairness better than PF scheduling, it performs worse from the perspective of total throughput.[1] Thus, it is not helpful for solving the fundamental problem of multicast systems that total throughput is critically reduced due to the dependency on the lowest channel gain. It will be numerically evaluated in more detail in section IV.

Based on [9], the PF scheduler has the following optimality property.

Theorem 1 ([9]): Under the PF algorithm with averaging time scale $t_c = \infty$, the long-term average throughput of each user exists almost surely, and the algorithm maximizes

$$\sum_{k=1}^{K} \log R_k = \log \left\{ \prod_{k=1}^{K} R_k \right\}. \tag{3}$$

Using Theorem 1 and assuming that available total transmit power is limited by P_T, the optimization RA problem for PF scheduling can be written as follows:

$$\begin{aligned} &\max_{c_n, \rho_{k,n}} \sum_{k=1}^{K} \log R_k = \max_{c_n, \rho_{k,n}} \sum_{k=1}^{K} \log \sum_{n=1}^{N} c_n \rho_{k,n} \\ &\text{subject to } \sum_{n=1}^{N} \max_k \left(\frac{f(c_n)\rho_{k,n}}{\alpha_{k,n}^2} \right) \le P_T. \end{aligned} \tag{4}$$

This problem is nonlinear because of the nonlinearity of $f(c)$ and *max* function. For example, in the case of $M-$ary quadrature amplitude modulation (M-QAM), $f(c)$ can be represented as $f(c) = \frac{N_o}{3}\left[Q^{-1}(p_e/4)\right]^2 (2^c - 1)$, where p_e is the required bit error rate (BER), $N_o/2$ denotes the variance of the additive white Gaussian noise (AWGN), and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$.

[1]By the property that all data rates are almost identical in max-min fairness scheduling, it is often employed in wireless or wireless communications where there are many links between transmitter and receiver.

III. Proposed Algorithm for PF Scheduling

Using the similar method in [1], we can convert the non-linear constraint in (4) into a number of linear constraints. If we use the fact that c_n takes only integer values and assume M-QAM modulation, $f(c)$ becomes constants as follows: $f(c_n) = \{0, f(1), \cdots, f(M)\}$. In order to make $f(c_n)$ integer variable, the new indicator $\gamma_{k,n,c}$ is defined as follows:

$$\gamma_{k,n,c} = \begin{cases} \rho_{k,n}, & c_n = c \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

Once c_n is used for the n-th subcarrier, $\gamma_{k,n,c}$ should be zero for other values except c_n. This constraint can be expressed as

$$\begin{aligned} &\left\{ \begin{array}{l} 1 \le \sum_{k=1}^{K} \gamma_{k,n,1} \le K \quad \text{and} \\ \sum_{c \ne 1} \sum_{k=1}^{K} \gamma_{k,n,c} = 0 \end{array} \right\} \text{or} \\ &\qquad \vdots \\ &\left\{ \begin{array}{l} 1 \le \sum_{k=1}^{K} \gamma_{k,n,M} \le K \quad \text{and} \\ \sum_{c \ne M} \sum_{k=1}^{K} \gamma_{k,n,c} = 0 \end{array} \right\}. \end{aligned} \tag{6}$$

Using $\gamma_{k,n,c}$ defined in Eq. (5), $c_n \rho_{k,n}$ and $f(c_n)\rho_{k,n}$ are given by $c_n \rho_{k,n} = \sum_{c=1}^{M} c \cdot \gamma_{k,n,c}$ and $f(c_n)\rho_{k,n} = \sum_{c=1}^{M} f(c)\gamma_{k,n,c}$, respectively. By converting *max* function into a set of linear equations, the non-linear constraint is converted into several linear equations. With these constraints, the optimization problem in Eq. (4) can be converted into the following:

$$\begin{aligned} &\max_{\gamma_{n,k,c}} \sum_{k=1}^{K} \log R_k = \max_{\gamma_{n,k,c}} \sum_{k=1}^{K} \log \sum_{n=1}^{N} \sum_{c=1}^{M} c \cdot \gamma_{k,n,c} \\ &\text{subject to } \sum_{n=1}^{N} \sum_{c=1}^{M} \frac{f(c)\gamma_{1,n,c}}{\alpha_{1,n}^2} \le P_T, \\ &\sum_{n=1}^{N-1} \sum_{c=1}^{M} \frac{f(c)\gamma_{1,n,c}}{\alpha_{1,n}^2} + \sum_{c=1}^{M} \frac{f(c)\gamma_{2,N,c}}{\alpha_{2,N}^2} \le P_T, \\ &\qquad \vdots \\ &\sum_{n=1}^{N} \sum_{c=1}^{M} \frac{f(c)\gamma_{K,n,c}}{\alpha_{K,n}^2} \le P_T, \\ &\text{and the constraints in Eq. (6)} \end{aligned} \tag{7}$$

However, this problem remains the nonlinear one because of *log* function in the object function. Since *log* function includes two summations for variable $\gamma_{k,n,c}$, it may be difficult to convert into linear equations. In addition, even if we can succeed in linear conversion through the elaborate effort, the above optimization remains a NP-hard problem requiring large complexity.

Therefore, we consider only a simple heuristic algorithm for solving PF subcarrier/bit allocation. In order to reduce the complexity, we adopt a two-step approach separating subcarrier allocation and bit loading. This approach has been employed in several literatures [1], [10]. Since bit loading algorithm is the same as the modified Levin-Campello algorithm in [1], we focus only on PF subcarrier allocation algorithm.

Based on Theorem 1, the subcarrier is allocated in a way that maximizes the product of date rates for all the users. The specific procedures are as follows:

1) At first we consider $\rho_{k,n}$, calculated in [1] for total throughput maximization, as the initial value. Based on this, we calculate the data rate of each user R_k and then obtain the product of data rates of all the users. The resultant product value is referred to as P_{old}.
2) Find the user index κ whose date rate is minimum. And then, for that user, select the subcarrier index n^* having the maximum channel gain among all the subcarriers where $\rho_{\kappa,n}$ is zero.

$$\kappa = \arg\min_{k} R_k. \tag{8}$$

$$n^* = \arg\max_{n} \alpha_{\kappa,n}^2, \text{for all } n \text{ satisfying } \rho_{\kappa,n} = 0. \tag{9}$$

3) Set the subcarrier indicator of κ-th user's n^*-th subcarrier into one. Renew the subcarrier indicator for the n^*-th subcarrier.

$$\rho_{\kappa,n^*} = 1. \tag{10}$$

$$\rho_{k,n^*} = \begin{cases} 1, & \alpha_{k,n^*}^2 \ge \alpha_{\kappa,n^*}^2 \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

4) Using $\rho_{k,n}$ obtained in step 3, calculate R_k for all the users and the new product value of P_{new}. If P_{new} is greater than P_{old}, replace P_{old} with P_{new} and then repeat the above steps from 2 to 4. Otherwise, stop the procedure.

$$\begin{aligned} &P_{new} > P_{old} \Rightarrow P_{old} = P_{new}, \\ &\qquad\qquad \text{and repeat step } 2 \sim 4, \\ &P_{new} \le P_{old} \Rightarrow \text{stop.} \end{aligned} \tag{12}$$

Now let us consider the specific reasons for performing each step. In the first step, by adopting the subcarrier allocation solution for maximizing total throughput as the initial value, we can strengthen the direction of keeping total throughput. Since this heuristic algorithm can fall into a local optimum, it is of great importance to determine the initial value of $\rho_{k,n}$. In the second and third step, by supporting the user having minimum data rate, the fairness can be strengthened. In addition, in order to reduce complexity without severe degradation of throughput, we employ a heuristic scheme shown in Eq. (9). That is, subcarrier index is quickly selected by finding the maximum channel gain. Finally, since PF scheduler maximizes the product of data rates based on Theorem 1, the product value of data rates is used as a stopping criterion. The efficacy of this simple heuristic algorithm will be numerically evaluated in the next section.

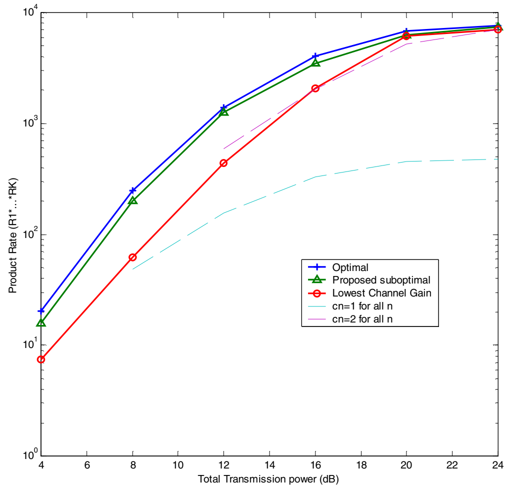

Fig. 2. Proportional Fairness: Comparison of the optimum solution and the proposed heuristic algorithm for $N = 4$, $K = 4$ and $M = 2$

IV. Numerical Results

In multicast OFDM systems, the proposed suboptimum algorithm for PF scheduling is compared with the optimum solution.[2] Next, for the case of large values of parameters, it is compared with that of the lowest channel gain (LCG) method, where all the subcarriers are shared by all the users and bits are loaded using the modified Levin-Campello algorithm. In the case of LCG method, only bit loading information is required at the receiver because all the users share the subcarrier. Finally, by comparing different scheduling algorithms, we show that PF scheduling is suitable in multicast OFDM systems from all the perspectives of throughput and fairness.

Simulations are performed under the following assumptions: the channel is a frequency selective Rayleigh fading channel with the equal gain profile; the required BER is $p_e = 10^{-4}$; the noise variance $N_o/2 = 1$; the number of users K is between two to 16. During the simulation, 100 independent channels are generated and the results in figures are the average of 100 trials.

A. Proportional Fairness Scheduling

Fig. 2 shows the comparison of the optimum solution and proposed heuristic algorithm when the number of subcarriers $N = 4$, maximum loaded bits $M = 2$, and the number of users is four. The performance gap is within about 5%. Compared to the LCG scheme and fixed modulation, e.g., $c_n = 1$ or $c_n = 2$, it can be said that the performance difference between the optimum and proposed algorithms is not large; thus it can be said that the proposed heuristic algorithm performs well. For large transmission power, it is observed that product value is saturated regardless of any type of algorithms because the maximum loaded bits are limited by two.

[2]The optimum solution was obtained through exhaustive search; hence, it is quite difficult to obtain the optimum solution for large values of parameters. For this reason, we consider only the case for small numbers of subchannels and users.

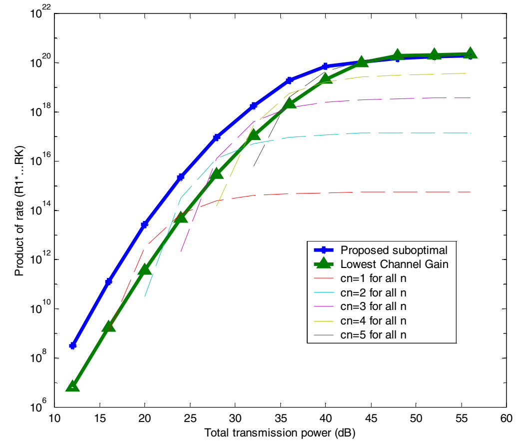

Fig. 3. Proportional Fairness: Product of data rates as a function of total transmission power P_T for $N = 64$, $K = 8$, and $M = 5$

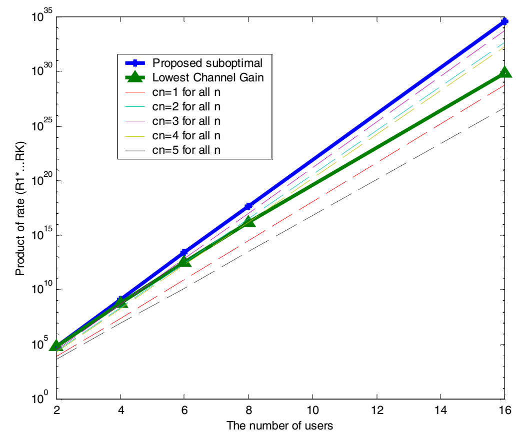

Fig. 4. Proportional Fairness: Product of data rates as a function of the number of users K for $N = 64$, $M = 5$, and $P_T = 30$ dB

To show the performance gain of the proposed heuristic algorithm in practical OFDM systems such as 802.11 [11], we consider the case of large parameters, e.g., $N = 64$, $M = 5$. In Fig. 3, product of data rate with varying total transmission power is shown when $K = 8$ and the number of channel taps is eight. For a wide range of transmission power, the product of data rates in the proposed algorithm is always greater than that in the LCG scheme. For large transmission power, however, the LCG scheme is slightly better. Nevertheless, since we are more interested in the case of insufficient transmission power in practical systems, the proposed heuristic scheme is meaningful. In Fig. 4, we can observe that the product of data rates in the proposed algorithm increases with the number of users. This is because much more information can be shared with the increase of users.

B. Comparison of Different Scheduling Algorithms

Fig. 5 shows the throughput of three scheduling algorithms, i.e., total throughput maximization, proportional fairness, and max-min fairness when the number of subcarriers is four, the number of users is four, and the number of maximum loaded bits is two. For max-min fairness, we formulated the optimization problems as

$$\arg\max_{\gamma_{k,n,c}} \min_{k} R_k = \arg\max_{\gamma_{k,n,c}} \min_{k} \sum_{c=1}^{M} \sum_{n=1}^{N} c \cdot \gamma_{k,n,c} \tag{13}$$

subject to the same constraints in Eq. (4).

The solution was obtained through exhaustive search. In this figure, the performance gap between total throughput maximization and PF is small compared to max-min fairness. In addition, the absolute throughput difference between total throughput maximization and PF is within about 5% for a practical range of total transmission power.

In order to evaluate the fairness of each scheduling scheme, we consider the variance of data rates of all the users as performance measure of fairness. Since we assume that in this simulation the average channel gain is the same for all the users, it could make sense to set the variance as performance measure of fairness. As shown in Fig. 6, the variance of max-min fairness is almost the same that of LCG method. That is, from the perspective of fairness, max-min fairness has the best performance among three scheduling schemes. PF scheduling has middle performance between total throughput maximization and max-min fairness.

Based on the above two observations, we can see that PF scheduling is a compromised technique to increase the fairness to some extent while minimizing throughput degradation. Thus PF scheduling may be the best solution for multicast OFDM systems if we consider both throughput and fairness with the same importance.

V. Conclusions

In this paper, the optimization problem for proportional fairness (PF) scheduling has been formulated in multicast OFDM systems under the assumption that hierarchical data can be combined in the receiver. To reduce the complexity for solving the optimum solution (NP-hard), we proposed the efficient heuristic algorithm separating subcarrier allocation and bit loading. Through the simulations, it was shown that the performance difference between the optimum and the proposed suboptimum algorithms was within about 5%. In addition, we have concluded that PF scheduling might be the most suitable for multicast OFDM systems if we consider both throughput and fairness.

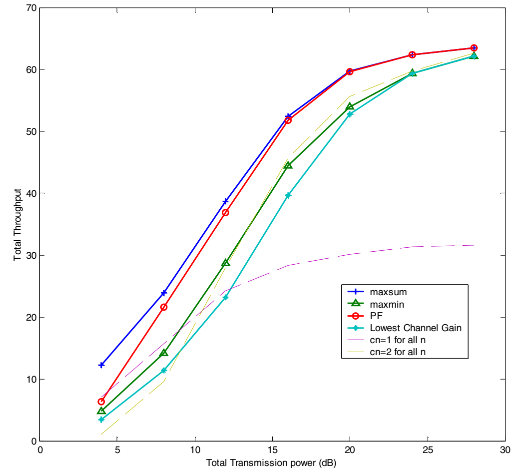

Fig. 5. Throughput comparison of three scheduling algorithms, i.e., total sum maximization, proportional fairness, and max-min fairness as a function of total transmission power for $N = 4$, $K = 4$, and $M = 2$

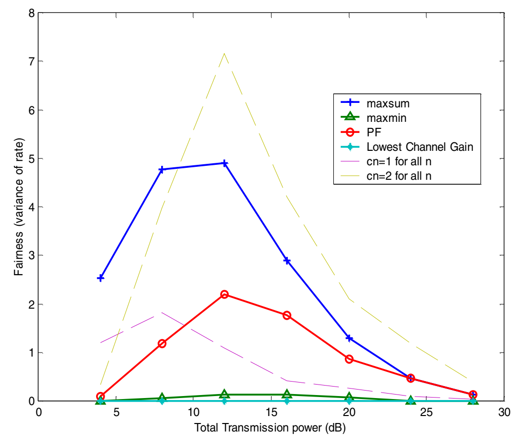

Fig. 6. Fairness comparison of three scheduling algorithms, i.e., total sum maximization, proportional fairness, and max-min fairness as a function of total transmission power for $N = 4$, $K = 4$, and $M = 2$

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