Degrees of Freedom of Uplink–Downlink Multiantenna Cellular Networks

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***Abstract*—An uplink–downlink cellular network is studied in which the first base station (BS) with M_1 antennas receives independent messages from its N_1 serving users, while the second BS with M_2 antennas transmits independent messages to its N_2 serving users. Each user is assumed to have a single antenna. Under this uplink–downlink setting, the sum degrees of freedom (DoF) is completely characterized as the minimum of $(N_1N_2 + \min(M_1, N_1)(N_1 - N_2)^+ + \min(M_2, N_2)(N_2 - N_1)^+)/\max(N_1, N_2)$, $M_1 + N_2, N_1 + M_2$, $\max(M_1, M_2)$, and $\max(N_1, N_2)$, where a^+ denotes $\max(0, a)$. The result demonstrates that, depending on the network configuration, operating one of the cells as uplink and the other cell as downlink can improve DoF compared to the conventional uplink or downlink operation, in which both cells operate as either uplink or downlink.**

I. Introduction

Characterizing the capacity of cellular networks is one of the fundamental problems in network information theory. Unfortunately, even for the simplest setting consisting of two base stations (BSs) having one serving user each, which is referred to as the two-user interference channel (IC), capacity is not completely characterized for general channel parameters [1], [2]. Exact capacity results being notoriously difficult to obtain, many researchers have recently studied approximate capacity characterizations in the shape of so-called "degrees of freedom (DoF)", which captures the behavior of capacity as the signal-to-noise ratio becomes large.

The DoF metric has received a great deal of attention and thoroughly analyzed as multiantenna techniques emerged [3], [4], especially in cellular networks [5]–[8] because of their potential to increase the DoF of cellular networks. Roughly speaking, equipping multiple antennas at the BS and/or users can drastically increase the sum DoF of single-cell cellular networks proportionally with the number of antennas.

Under multicell environment, Cadambe and Jafar recently made a remarkable progress showing that the sum DoF for the K-user IC is given by $K/2$ [9], which corresponds to the K-cell cellular network having one serving user in each cell. A new interference mitigation paradigm called signal space interference alignment (IA) has been proposed to achieve the sum DoF $K/2$ [9]. Different IA schemes have been also developed under the name of signal scale IA [10], [11] and ergodic IA [12], [13]. Multicell cellular networks having multiple serving users at each cell has been studied in [14], [15] under both uplink and downlink operation, each of which is called interfering multiple access channel (IMAC) [14] and interfering broadcast channel (IBC) [14], [15]. It was shown in [14], [15] that multiple users in each cell is beneficial for increasing the sum DoF of IMAC and IBC by utilizing multiple users in each cell for IA.

As a natural extension, integrating multiantenna techniques and IA techniques has been recently studied to boost the DoF of multicell multiantenna cellular networks. The DoF of the K-user IC having M antennas at each transmitter and N antennas at each receiver has been analyzed in [16]. More recently, the IMAC and IBC models have been extended to multiantenna BS and/or multiantenna users, see [17]–[20] and the references therein.

In this paper, we study a multiantenna cellular network in which the first and second cells operate as uplink and downlink respectively. This uplink–downlink model is motivated to figure out whether operating only as conventional uplink or downlink is optimal or not in terms of the DoF for multicell multiantenna cellular networks. Notice that recent works on multiantenna IMAC and IBC cannot provide an answer for this fundamental question since it inherently assumes the conventional uplink or downlink operation. We completely characterize the sum DoF of uplink–downlink multiantenna cellular networks and show that, depending on the network configuration, uplink–downlink operation is beneficial for increasing the sum DoF compared to the conventional uplink or downlink operation.

II. Problem Formulation

Throughout the paper, $[1:n]$ denotes $\{1, 2, \cdots, n\}$, $\mathbf{0}_n$ denotes the $n \times 1$ all-zero vector, and $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. For a real value a, a^+ denotes $\max(0, a)$. For a set of vectors $\{\mathbf{a}_i\}$, $\text{span}(\{\mathbf{a}_i\})$ denotes the signal space spanned by the vectors in $\{\mathbf{a}_i\}$. For a matrix $\mathbf{A}$, $\mathbf{A}^\dagger$ denotes the transpose of $\mathbf{A}$. For a set of matrices $\{\mathbf{A}_i\}$, $\text{diag}(\mathbf{A}_1, \cdots, \mathbf{A}_n)$ denotes the block diagonal matrix consisting of $\{\mathbf{A}_i\}$.

A. Uplink–Downlink Multiantenna Cellular Networks

Consider a multiantenna cellular network depicted in Fig. 1 in which the first cell operates as uplink and the second

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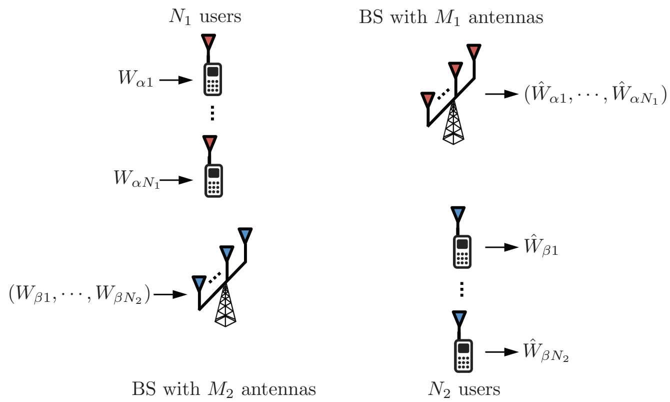

Fig. 1. Uplink–downlink multiantenna cellular networks.

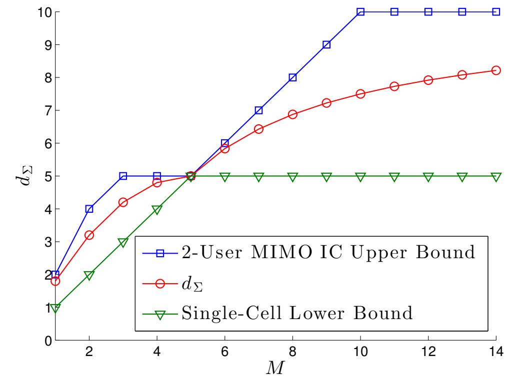

Fig. 2. d_Σ in Theorem 1 with respect to M when $N = 5$, where $M_1 = N_2 = M$ and $M_2 = N_1 = N$.

cell operates as downlink. Specifically, the BS in the first cell equipped with M_1 antennas wishes to receive an independent message $W_{\alpha i}$ from the ith user in the same cell for all $i \in [1 : N_1]$. On the other hand, the BS in the second cell equipped with M_2 antennas wishes to send an independent message $W_{\beta j}$ to the jth user in the same cell for all $j \in [1 : N_2]$. Each user is assumed to have a single antenna.

The $M_1 \times 1$ received signal vector of the first BS at time t is given by

$$\mathbf{y}_\alpha[t] = \sum_{i=1}^{N_1} \mathbf{h}_{\alpha i}[t] x_{\alpha i}[t] + \mathbf{G}_\alpha[t] \mathbf{x}_\beta[t] + \mathbf{z}_\alpha[t] \quad (1)$$

and the received signal of the jth user in the second cell at time t is given by

$$y_{\beta j}[t] = \mathbf{h}_{\beta j}[t] \mathbf{x}_\beta[t] + \sum_{i=1}^{N_1} g_{\beta j i}[t] x_{\alpha i}[t] + z_{\beta j}[t], \quad (2)$$

where $j \in [1 : N_2]$. Here $\mathbf{h}_{\alpha i}[t]$ is the $M_1 \times 1$ channel vector, $\mathbf{G}_\alpha[t]$ is the $M_1 \times M_2$ channel matrix, $\mathbf{h}_{\beta j}[t]$ is the $1 \times M_2$ channel vector, and $g_{\beta j i}[t]$ is the scalar channel. The additive noises $\mathbf{z}_\alpha[t]$ and $z_{\beta j}[t]$ are assumed to follow $\mathcal{N}(\mathbf{0}_{M_1}, \mathbf{I}_{M_1})$ and $\mathcal{N}(0, 1)$, respectively. Each user and BS should satisfy the average power constraint P, i.e., $E(x_{\alpha i}^2[t]) \le P$ for all $i \in [1 : N_1]$ and $E(\|\mathbf{x}_\beta[t]\|^2) \le P$.

We assume that all channel coefficients are independent and identically distributed (i.i.d.) drawn from a continuous distribution and vary independently over each time slot. Global channel state information is assumed to be available at each user and BS.

B. Degrees of Freedom

Let $W_{\alpha i}$ and $W_{\beta j}$ be chosen uniformly at random from $[1 : 2^{nR_{\alpha i}}]$ and $[1 : 2^{nR_{\beta j}}]$ respectively, where $i \in [1 : N_1]$ and $j \in [1 : N_2]$. A rate tuple $(R_{\alpha 1}, \cdots, R_{\alpha N_1}, R_{\beta 1}, \cdots, R_{\beta N_2})$ is said to be achievable if there exists a sequence of $(2^{nR_{\alpha 1}}, \cdots, 2^{nR_{\alpha N_1}}, 2^{nR_{\beta 1}}, \cdots, 2^{nR_{\beta N_2}}; n)$ codes such that $\Pr(\hat{W}_{\alpha i} \neq W_{\alpha i}) \to 0$ and $\Pr(\hat{W}_{\beta j} \neq W_{\beta j}) \to 0$ as n increases for all $i \in [1 : N_1]$ and $j \in [1 : N_2]$. Then the achievable sum DoF is given by

$$\lim_{P \to \infty} \frac{\sum_{i=1}^{N_1} R_{\alpha i} + \sum_{j=1}^{N_2} R_{\beta j}}{\frac{1}{2} \log P}. \quad (3)$$

For notational convenience, denote the maximum achievable sum DoF by d_Σ.

III. Main Result

In this section, we state our main result. We completely characterize d_Σ in the following theorem.

Theorem 1: For the uplink–downlink multiantenna cellular network, d_Σ is given by

$$\begin{aligned} \min\{&(N_1 N_2 + \min(M_1, N_1)(N_1 - N_2)^+ \\ &+ \min(M_2, N_2)(N_2 - N_1)^+)/\max(N_1, N_2), \\ &M_1 + N_2, N_1 + M_2, \max(M_1, M_2), \max(N_1, N_2)\}. \end{aligned} \quad (4)$$

Proof: The proof outline for the achievability is in Section IV and we refer to the full paper in [21] for the detailed proof. The converse proof is in Section V. ■

Example 1 (Symmetric Case): Consider the following three symmetric settings:

- Case A: $M_1 = N_1 := M$ and $M_2 = N_2 := N$,
- Case B: $M_1 = M_2 := M$ and $N_1 = N_2 := N$,
- Case C: $M_1 = N_2 := M$ and $M_2 = N_1 := N$.

For Cases A and B, d_Σ is given by $\max\{M, N\}$ and $\min\{M, N\}$ respectively, which is trivially achievable by operating one of the two cells. The two-user multiple input multiple output (MIMO) IC upper bound in [22], which corresponds to the model that allows full cooperation between the users within each cell, is tight for these two cases. For Case C, Theorem 1 shows that

$$d_\Sigma = \begin{cases} \frac{M(2N-M)}{N} & \text{if } M \le N, \\ \frac{N(2M-N)}{M} & \text{if } M \ge N. \end{cases} \quad (5)$$

Figure 2 plots (5) when $N = 5$. For comparison, we also plot the two-user MIMO IC upper bound and the single-cell lower bound, each of which is given by $\min\{2M, 2N, \max(M, N)\}$ and $\min(M, N)$ respectively. Note that (5) is not trivially

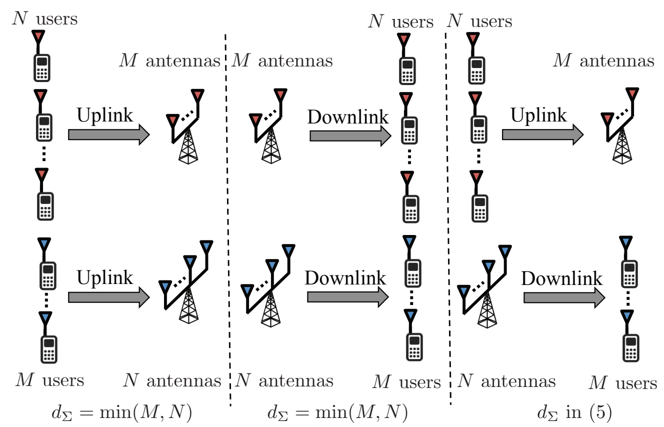

Fig. 3. Sum DoF for uplink, downlink, and uplink–downlink operation for $M_1 = N_2 = M$ and $M_2 = N_1 = N$.

achievable and, moreover, the two-user MIMO IC upper bound is not tight for all M and N satisfying $M \neq N$.

Remark 1 (Dof Gain From Uplink–Downlink Operation): Theorem 1 demonstrates that, depending on the network configuration, operating unlink and downlink simultaneously between cells can improve d_Σ compared to the conventional operation in which the entire cells operate as either uplink or downlink. For instance, consider the network configuration in Case C of Example 1, see Fig. 3. If we operate both cells either uplink or downlink, then d_Σ is upper bounded by the single-cell lower bound, i.e., $\min(M, N)$. On the other hand, uplink–downlink operation achieves (5), which is strictly larger than $\min(M, N)$ for any $M \neq N$. The DoF gain from uplink–downlink operation is discussed in more details over a four-parameter space (M_1, M_2, N_1, N_2) in Section VI in [21].

Remark 2 (User Cooperation): From Theorem 1, d_Σ is given by $\frac{2N_1-1}{N_1}$ for $M_2 = 2$ and $M_1 = N_2 = 1$, which converges to two as N_1 increases. Recall that, for the two-cell IBC, the number of users in both cells should tend to infinity in order to achieve the interference-free sum DoF of two [14], [15]. Hence this simple example shows that, if user cooperation is allowed only for one of the two cells, cooperation between two users is enough to achieve $d_\Sigma \to 2$ if the number of users in the other cell tends to infinity. In this sense, one-side user cooperation is still powerful for boosting DoF.

IV. Achievability

In this section, we first explain the main achievability idea based on a simple example and then briefly introduce two proposed schemes.

A. Main Idea

For better understanding, we briefly explain the main idea here assuming that $M_2 = 2$, $M_1 = N_2 = 1$. Figure 4 illustrates how to achieve $d_\Sigma = \frac{2N_1-1}{N_1}$ for this case. Communication takes place via transmit beamforming over a block of N_1 time slots. Denote $\text{diag}(\mathbf{h}_{\alpha i}[1], \cdots, \mathbf{h}_{\alpha i}[N_1])$, $\text{diag}(\mathbf{h}_{\beta 1}[1], \cdots, \mathbf{h}_{\beta 1}[N_1])$, $\text{diag}(\mathbf{G}_\alpha[1], \cdots, \mathbf{G}_\alpha[N_1])$, and $\text{diag}(g_{\beta 1 i}[1], \cdots, g_{\beta 1 i}[N_1])$ by $\bar{\mathbf{H}}_{\alpha i}$, $\bar{\mathbf{H}}_{\beta 1}$, $\bar{\mathbf{G}}_\alpha$, and $\bar{\mathbf{G}}_{\beta 1 i}$ respectively, where $i \in [1 : N_1]$. The ith user in the first cell transmits a single stream via the $N_1 \times 1$ beamforming vector $\bar{\mathbf{v}}_{\alpha i}$, where $i \in [1 : N_1]$. The BS in the second cell transmits $N_1 - 1$ streams via $2N_1 \times 1$ beamforming vectors $\{\bar{\mathbf{v}}_{\beta j}\}_{j \in [1:N_1-1]}$. Then we can set linearly independent $\{\bar{\mathbf{v}}_{\alpha i}\}_{i \in [1:N_1]}$ satisfying the uplink inter-cell IA condition, i.e., $\bar{\mathbf{G}}_{\beta 1 i} \bar{\mathbf{v}}_{\alpha i}$ is the same for all $i \in [1 : N_1]$. We can also set linearly independent $\{\bar{\mathbf{v}}_{\beta j}\}_{j \in [1:N_1-1]}$ satisfying the downlink inter-cell interference nulling (IN) condition, i.e., $\bar{\mathbf{G}}_\alpha \bar{\mathbf{v}}_{\beta j} = \mathbf{0}_{N_1}$ for all $i \in [1 : N_1 - 1]$. Since total $2N_1 - 1$ streams are delivered over N_1 time slots, $d_\Sigma = \frac{2N_1-1}{N_1}$ is achievable.

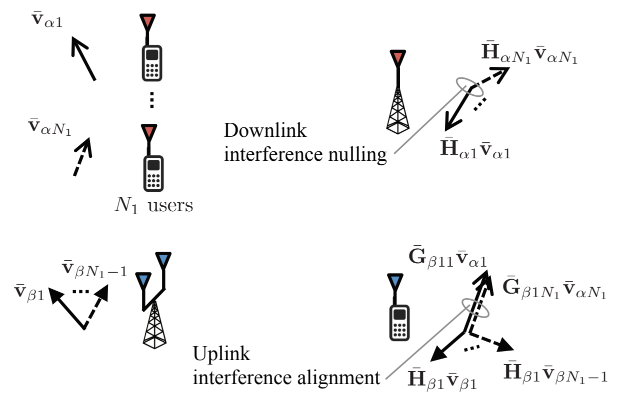

Fig. 4. d_Σ-achievable transmit beamforming for $M_2 = 2$, $M_1 = N_2 = 1$.

In the following two subsections, we will introduce two IA–IN schemes for general M_1, M_2, N_1, and N_2. As shown in Fig. 4, the first key ingredient follows uplink inter-cell IA from the users in the first cell to the users in the second cell. Unlike the simple case in Fig. 4, asymptotic IA using an arbitrarily large number of time slots is generally needed for simultaneously aligning interference from multiple transmitters at multiple receivers [9]. The second key ingredient follows downlink inter-cell and intra-cell IN by multiantenna transmit beamforming from the BS in the second cell to the BS in the first cell and the users in the same cell.

B. Uplink Inter-Cell IA and Downlink Inter-Cell and Intra-Cell IN

The first IA–IN scheme is the extension of the scheme in Section IV-A, which achieves d_Σ if $M_1 \leq M_2$. Assume an arbitrarily large number of time slots denoted by T. Communication takes place via transmit beamforming over a block of T time slots. Denote $\text{diag}(\mathbf{h}_{\alpha i}[1], \cdots, \mathbf{h}_{\alpha i}[T])$, $\text{diag}(\mathbf{h}_{\beta j}[1], \cdots, \mathbf{h}_{\beta j}[T])$, $\text{diag}(\mathbf{G}_\alpha[1], \cdots, \mathbf{G}_\alpha[T])$, and $\text{diag}(g_{\beta j i}[1], \cdots, g_{\beta j i}[T])$ by $\bar{\mathbf{H}}_{\alpha i}$, $\bar{\mathbf{H}}_{\beta j}$, $\bar{\mathbf{G}}_\alpha$, and $\bar{\mathbf{G}}_{\beta j i}$ respectively, where $i \in [1 : N_1]$ and $j \in [1 : N_2]$. Figure 5 illustrates the proposed IA–IN scheme, where $0 \leq \lambda_1, \lambda_2 \leq 1$. Due to page limitation, we briefly explain it in this paper and refer to [21] for the detailed description.

Uplink inter-cell IA: For $i \in [1 : N_1]$, the ith user in the first cell transmits $\lambda_1 T(1 - \epsilon)$ streams via $T \times 1$ beamforming vectors $\{\bar{\mathbf{v}}_{\alpha i}^{(k)}\}_{k \in [1:\lambda_1 T(1-\epsilon)]}$. From signal space IA [9], we

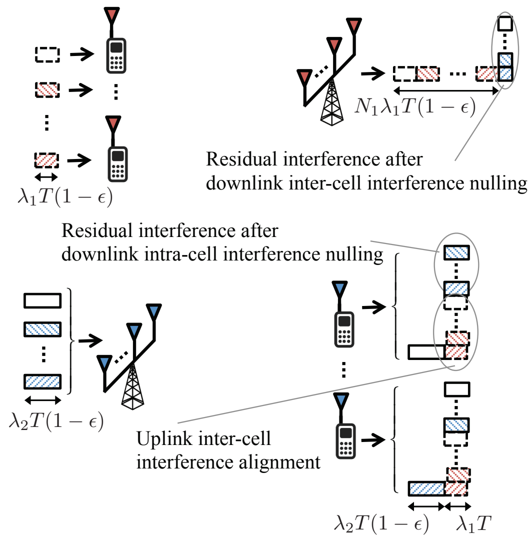

Fig. 5. Uplink inter-cell IA and downlink inter-cell and intra-cell IN, where for convenience we assume $\lambda_1 \le \lambda_2$ in the figure.

can set linearly independent $\{\bar{\mathbf{v}}_{\alpha i}^{(k)}\}_{k\in[1:\lambda_1 T(1-\epsilon)]}$ for all $i \in [1 : N_1]$ such that

$$\text{span}\left(\{\bar{\mathbf{G}}_{\beta ji}\bar{\mathbf{v}}_{\alpha i}^{(k)}\}_{i\in[1:N_1],j\in[1:N_2],k\in[1:\lambda_1 T(1-\epsilon)]}\right) \quad (6)$$

occupies at most $\lambda_1 T$ dimensional subspace in T dimensional signal space almost surely and $\epsilon \to 0$ as $T \to \infty$.

Downlink inter-cell and intra-cell IN: For $j \in [1 : N_2]$, the BS in the second cell transmits $\lambda_2 T(1-\epsilon)$ streams to the jth user in the same cell via $M_2 T \times 1$ beamforming vectors $\{\bar{\mathbf{v}}_{\beta j}^{(l)}\}_{l\in[1:\lambda_2 T(1-\epsilon)]}$. Hence, in order to nullify inter-cell interference,

$$\left(\bar{\mathbf{H}}_{\alpha i}\bar{\mathbf{v}}_{\alpha i}^{(k)}\right)^{\dagger}\bar{\mathbf{G}}_{\alpha}\bar{\mathbf{v}}_{\beta j}^{(l)} = 0 \quad (7)$$

should be satisfied for all $i \in [1 : N_1]$, $j \in [1 : N_2]$, $k \in [1 : \lambda_1 T(1-\epsilon)]$, and $l \in [1 : \lambda_2 T(1-\epsilon)]$. Similarly, to nullify intra-cell interference,

$$\left(\bar{\mathbf{H}}_{\beta i}\bar{\mathbf{v}}_{\beta i}^{(k)}\right)^{\dagger}\bar{\mathbf{H}}_{\beta i}\bar{\mathbf{v}}_{\beta j}^{(l)} = 0 \quad (8)$$

should be satisfied for all $i, j \in [1 : N_2]$ with $i \neq j$ and $k, l \in [1 : \lambda_2 T(1-\epsilon)]$. Therefore, from (7) and (8), $\bar{\mathbf{v}}_{\beta j}^{(l)}$ should be orthogonal with the vectors in

$$\begin{aligned}&\{\bar{\mathbf{v}}_{\alpha i}^{(k)\dagger}\bar{\mathbf{H}}_{\alpha i}^{\dagger}\bar{\mathbf{G}}_{\alpha}\}_{i\in[1:N_1],k\in[1:\lambda_1 T(1-\epsilon)]},\\ &\{\bar{\mathbf{v}}_{\beta i}^{(k)\dagger}\bar{\mathbf{H}}_{\beta i}^{\dagger}\bar{\mathbf{H}}_{\beta i}\}_{i\in[1:N_2],i\neq j,k\in[1:\lambda_2 T(1-\epsilon)]}.\end{aligned} \quad (9)$$

Since there are total $(N_1\lambda_1 + (N_2-1)\lambda_2)T(1-\epsilon)$ vectors in (9) and $\bar{\mathbf{v}}_{\beta j}^{(l)}$ has $M_2 T$ elements, we can set linearly independent $\{\bar{\mathbf{v}}_{\beta j}^{(l)}\}_{l\in[1:\lambda_2 T(1-\epsilon)]}$ orthogonal with the vectors in (9) for all $j \in [1 : N_2]$ if

$$M_2 T - (N_1\lambda_1 + (N_2-1)\lambda_2)T(1-\epsilon) > \lambda_2 T(1-\epsilon). \quad (10)$$

C. Uplink Inter-Cell IA and Downlink Intra-Cell IN

The second IA–IN scheme is a simple modification of the first IA–IN scheme. For downlink beamforming, $\{\bar{\mathbf{v}}_{\beta j}^{(l)}\}_{l\in[1:\lambda_2 T(1-\epsilon)]}$ is set only for intra-cell IN, but not for inter-cell IN. That is, (8) should be satisfied for all $i, j \in [1 : N_2]$ with $i \neq j$ and $k, l \in [1 : \lambda_2 T(1-\epsilon)]$, which yields

$$M_2 T - (N_2-1)\lambda_2 T(1-\epsilon) > \lambda_2 T(1-\epsilon) \quad (11)$$

instead of (10). As shown in (11), this scheme relaxes the downlink beamforming constraint, which requires less transmit antennas at the second BS but at the same time requires more receive antennas at the first BS due to no downlink inter-cell IN. The second IA–IN scheme achieves d_Σ if $M_1 \ge M_2$.

D. Achievable Sum DoF

Consider the first IA–IN scheme (see Fig. 5). In order for the users in the second cell to decode their intended streams, $\lambda_2 T(1-\epsilon) + \lambda_1 T \le T$ should be obviously satisfied. As the same reason, $N_1\lambda_1 T(1-\epsilon) \le M_1 T$ should be satisfied for successful decoding at the first BS. Lastly (10) should be satisfied for downlink inter-cell and intra-cell IN. Upon the above three conditions, the sum DoF $\frac{N_1\lambda_1 T(1-\epsilon)+N_2\lambda_2 T(1-\epsilon)}{T}$ is achievable. Therefore, from the fact that $\epsilon \to 0$ as $T \to \infty$, the achievable sum DoF of the first IA–IN scheme is given by

$$\max_{\substack{\lambda_1+\lambda_2\le 1\\ N_1\lambda_1\le M_1\\ N_1\lambda_1+N_2\lambda_2\le M_2}} \{N_1\lambda_1 + N_2\lambda_2\}. \quad (12)$$

Now consider the second IA–IN scheme. Since downlink inter-cell IN is not applied for the second IA–IN scheme, the second and third conditions of the first IA–IN scheme are changed to $N_1\lambda_1 T(1-\epsilon) + N_2\lambda_2 T(1-\epsilon) \le M_1 T$ and (11) respectively in this case. Therefore, the achievable sum DoF of the second IA–IN scheme is given by

$$\max_{\substack{\lambda_1+\lambda_2\le 1\\ N_1\lambda_1+N_2\lambda_2\le M_1\\ N_2\lambda_2\le M_2}} \{N_1\lambda_1 + N_2\lambda_2\}. \quad (13)$$

Let $d_{\Sigma,1}$ and $d_{\Sigma,2}$ denote the solutions of the two linear programmings in (12) and (13), respectively. Then we can show that $d_{\Sigma,1} = d_\Sigma$ if $M_1 \le M_2$ and $d_{\Sigma,2} = d_\Sigma$ if $M_1 \ge M_2$. We refer to Lemma 1 in [21] for the proof. Therefore, d_Σ in Theorem 1 is achievable.

V. Converse

In this section, we show the converse of Theorem 1. If we allow full cooperation within the N_1 users in the first cell and the N_2 users in the second cell, then the network becomes the two-user MIMO interference channel. Hence, from the result in [22], $d_\Sigma \le \min\{M_1 + N_2, N_1 + M_2, \max(M_1, M_2), \max(N_1, N_2)\}$. Then the remaining non-trivial part is to prove the first d_Σ constraint in (4).

Let $d_{\alpha i}$, $i \in [1 : N_1]$, denote the achievable DoF of the ith user in the first cell and $d_{\beta j}$, $j \in [1 : N_2]$, denote the achievable DoF of the jth user in the second cell. We first introduce the following lemma.

Lemma 1: For any $i \in [1 : N_1]$ and $j \in [1 : N_2]$

$$d_{\alpha i} + d_{\beta j} \leq 1. \tag{14}$$

Proof: Without loss of generality, it suffices to consider $i = j = 1$. Define $\bar{W}_{\alpha 1} = (W_{\alpha 2}, \cdots, W_{\alpha N_1})$, $\bar{W}_{\beta 1} = (W_{\beta 2}, \cdots, W_{\beta N_2})$, and $W_\beta = (W_{\beta 1}, \cdots, W_{\beta N_2})$. Let $\mathbf{y}_\alpha^n = (\mathbf{y}_\alpha[1], \cdots, \mathbf{y}_\alpha[n])$ and $y_{\beta 1}^n = (y_{\beta 1}[1], \cdots, y_{\beta 1}[n])$. Starting with Fano's inequality, we have

$$\begin{aligned} n(R_{\alpha 1} - \epsilon_n) &\leq I(W_{\alpha 1}; \mathbf{y}_\alpha^n) \\ &\leq I(W_{\alpha 1}; \mathbf{y}_\alpha^n, y_{\beta,1}^n, \bar{W}_{\alpha 1}, W_\beta) \\ &= I(W_{\alpha 1}; \mathbf{y}_\alpha^n, y_{\beta,1}^n | \bar{W}_{\alpha 1}, W_\beta). \end{aligned} \tag{15}$$

On the other hand,

$$\begin{aligned} n(R_{\beta 1} - \epsilon_n) &\leq I(W_{\beta 1}; y_{\beta 1}^n) \\ &\leq I(W_{\beta 1}; y_{\beta 1}^n, \bar{W}_{\alpha 1}, \bar{W}_{\beta 1}) \\ &= I(W_{\beta 1}; y_{\beta 1}^n | \bar{W}_{\alpha 1}, \bar{W}_{\beta 1}) \\ &= I(W_{\alpha 1}, W_{\beta 1}; y_{\beta 1}^n | \bar{W}_{\alpha 1}, \bar{W}_{\beta 1}) \\ &\quad - I(W_{\alpha 1}; y_{\beta 1}^n | \bar{W}_{\alpha 1}, W_\beta). \end{aligned} \tag{16}$$

From (15) and (16)

$$\begin{aligned} & n(R_{\alpha 1} + R_{\beta 1} - 2\epsilon_n) \\ & \leq I(W_{\alpha 1}, W_{\beta 1}; y_{\beta 1}^n | \bar{W}_{\alpha 1}, \bar{W}_{\beta 1}) \\ & + I(W_{\alpha 1}; \mathbf{y}_\alpha^n | \bar{W}_{\alpha 1}, W_\beta, y_{\beta 1}^n) \\ & \leq \sum_{i=1}^n I(\mathbf{x}_\beta[i], x_{\alpha 1}[i], \cdots, x_{\alpha N_1}[i]; y_{\beta 1}[i]) \\ & + I(W_{\alpha 1}; \mathbf{y}_\alpha^n | \bar{W}_{\alpha 1}, W_\beta, y_{\beta 1}^n) \end{aligned} \tag{17}$$

and

$$\begin{aligned} & I(W_{\alpha 1}; \mathbf{y}_\alpha^n | \bar{W}_{\alpha 1}, W_\beta, y_{\beta 1}^n) \\ & = h(\mathbf{y}_\alpha^n | \bar{W}_{\alpha 1}, W_\beta, y_{\beta 1}^n) - h(\mathbf{z}_\alpha^n) \\ & = \sum_{i=1}^n h(\mathbf{y}_\alpha[i] | \mathbf{y}_\alpha^{i-1}, \bar{W}_{\alpha 1}, W_\beta, y_{\beta 1}^n, \mathbf{x}_\beta[i], \\ & \qquad g_{\beta 11}[i] x_{\alpha 1}[i] + z_{\beta 1}[i], x_{\alpha 2}[i], \cdots, x_{\alpha N_1}[i]) - h(\mathbf{z}_\alpha^n) \\ & \leq \sum_{i=1}^n h\left(\mathbf{z}_\alpha[i] - \frac{\mathbf{h}_{\alpha 1}[i]}{g_{\beta 11}[i]} z_{\beta 1}[i]\right) - h(\mathbf{z}_\alpha^n) \\ & = n\, \mathrm{o}(P) \end{aligned} \tag{18}$$

Finally we have $d_{\alpha 1} + d_{\beta 1} \leq 1$ from (17) and (18), which completes the proof. ■

Adding (14) in Lemma 1 for all $i \in [1 : N_1]$ and $j \in [1 : N_2]$ provides

$$N_2 \sum_{i=1}^{N_1} d_{\alpha i} + N_1 \sum_{j=1}^{N_2} d_{\beta j} \leq N_1 N_2. \tag{19}$$

Obviously,

$$(N_1 - N_2)^+ \sum_{i=1}^{N_1} d_{\alpha i} \leq (N_1 - N_2)^+ \min(M_1, N_1). \tag{20}$$

$$(N_2 - N_1)^+ \sum_{j=1}^{N_2} d_{\beta j} \leq (N_2 - N_1)^+ \min(M_2, N_2), \tag{21}$$

Finally adding (19) to (21) yields the first d_Σ constraint in (4). Therefore, d_Σ is upper bounded by (4), which completes the converse proof.

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