Network Decomposition for Function Computation

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***Abstract*—We develop a network-decomposition framework to provide elementary parallel subnetworks that can constitute an original network without loss of optimality. In our earlier work, a network decomposition is constructed for the Avestimehr-Diggavi-Tse deterministic network which well captures key properties of wireless Gaussian networks. In this work, we apply this decomposition framework to general problem settings where receivers intend to compute *functions* of the messages generated at transmitters. Depending on functions, these settings include a variety of network problems, ranging from classical communication problems (such as multiple-unicast and multicast problems) to function computation problems. For many of these problems, we show that coding separately over the decomposed orthogonal subnetworks provides optimal performances, thus establishing a separation principle.**

I. Introduction

Communication networks are typically designed based on a separation approach where the whole task is divided into various modules and each module can then be independently designed. This separation methodology has been advocated due to its engineering advantages. For certain scenarios such as point-to-point source-channel coding problems [1], this approach also provides an optimal communication architecture although it does not guarantee the optimal performance in general.

In this work, we intend to develop a separation principle in the context of wireless networks which ensures the optimality of separate tasks independently performed across decomposed orthogonal modules. As an initial effort, we consider a simple abstraction model of wireless networks: the Avestimehr-Diggavi-Tse (ADT) deterministic network which well captures superposition and broadcast properties of wireless Gaussian networks [2].

In our earlier work [3], we developed a network decomposition for a two-transmitter two-receiver ADT network which enables splitting an original network into several orthogonal subnetworks. In [3], the network decomposition was investigated for a certain network scenario in which both receivers want to compute a linear function (modulo-2 sum) of Bernoulli sources generated at the transmitters. Interestingly, it was shown that in the scenario, coding separately over the decomposed orthogonal components achieves the optimal performance, i.e., the decomposition holds without loss of optimality. Subsequently in [4] it was also shown that the decomposition is optimal even in the presence of feedback.

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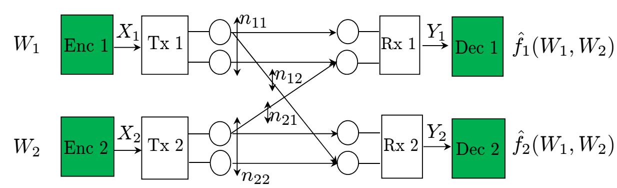


Fig. 1. Two-transmitter two-receiver Avestimehr-Diggavi-Tse (ADT) deterministic network.

Our interest is to examine the optimality of the network decomposition for more general scenarios in which receivers wish to compute arbitrary functions of the messages. Notice that depending on functions, these settings encompass a variety of network problems ranging from classical communication problems – such as multiple-unicast and multicast problems – to function computation problems. In this work, we show that the network decomposition is optimal also for two additional scenarios. The first scenario represents the two-unicast problem where each receiver wants to decode a message from its corresponding transmitter. The second scenario indicates the classical multicast problem in which both receivers want to decode all the messages of transmitters. Moreover, we establish the separation principle even in the presence of feedback. This result has potential to provide significant ramifications for the design of structured computation codes [5], [6] in wireless networks.

II. Model

We consider a two-transmitter two-receiver ADT deterministic network as depicted in Fig. 1. This network is described by four integer parameters n_{ij} which indicates the number of signal bit levels from transmitter i $(i = 1, 2)$ to receiver j $(j = 1, 2)$. Let $X_\ell \in \mathbb{F}_2^q$ be transmitter ℓ's encoded signal where $q = \max_{ij} n_{ij}$. The received signals are then given by

$$\begin{aligned} Y_1 &= \mathbf{G}^{q-n_{11}} X_1 \oplus \mathbf{G}^{q-n_{21}} X_2, \\ Y_2 &= \mathbf{G}^{q-n_{12}} X_1 \oplus \mathbf{G}^{q-n_{22}} X_2, \end{aligned} \tag{1}$$

where $\mathbf{G}$ is the q-by-q shift matrix, i.e., $[\mathbf{G}]_{ij} = \mathbf{1}\{i = j+1\}$ $(1 \le i \le q; 1 \le j \le q)$, and operations are performed in $\mathbb{F}_2$. We focus on a simple setting where $n := n_{11} = n_{22}$ and $m := n_{12} = n_{21}$. We denote this network by (m, n) model. Receiver ℓ wishes to compute a function $f_\ell(W_1, W_2)$ of the two independent messages W_1 and W_2, generated at the two transmitters, with N uses of the network, $\ell = 1, 2$.

We consider two cases, depending on the presence of feedback. In the nonfeedback case, the encoded signal $X_{\ell i}$ of transmitter ℓ at time i is a function of its own message W_ℓ. In the feedback case, on the other hand, $X_{\ell i}$ is a function of W_ℓ and past feedback signals from both receivers (Y_1^{i-1}, Y_2^{i-1}). Here we use shorthand notation to indicate the sequence up to $i-1$, e.g., $Y_1^{i-1} := (Y_{11}, \cdots, Y_{1(i-1)})$. Receiver ℓ uses a decoding function d_ℓ to estimate $f_\ell(W_1, W_2)$ from its received signal Y_ℓ^N. An error occurs whenever $d_\ell \neq f_\ell(W_1, W_2)$. The average probabilities of error are given by $\lambda_\ell = \mathbb{E}[P(d_\ell \neq f_\ell(W_1, W_2))]$, $\ell = 1, 2$.

We say that a rate pair (R_1, R_2) is achievable if there exists a family of codebooks and encoder/decoder functions such that the average decoding error probabilities go to zero as code length N tends to infinity. The capacity region $\mathcal{C}$ is the closure of the set of the achievable rate pairs. The symmetric capacity is defined as $C_{\mathsf{sym}} := \sup\{R : (R, R) \in \mathcal{C}\}$.

Depending on the functions, we consider three scenarios: (1) the function multicast problem considered in [3], [4]; (2) the two-unicast problem; (3) the classical multicast problem:

$$\begin{aligned}
&(\mathbf{S1}):\ f_1(W_1, W_2) = f_2(W_1, W_2) = W_1 \oplus W_2;\\
&(\mathbf{S2}):\ f_1(W_1, W_2) = W_1, f_2(W_1, W_2) = W_2;\\
&(\mathbf{S3}):\ f_1(W_1, W_2) = f_2(W_1, W_2) = (W_1, W_2).
\end{aligned}$$

In the first scenario, we consider a binary sequence for messages: $W_\ell = (S_{\ell 1}, \cdots, S_{\ell K})$, where $S_{\ell i}$'s are independent and identically distributed Bernoulli sources with $\mathsf{Bern}(\frac{1}{2})$.

III. Network Decomposition

We state the network decomposition established in the context of the first scenario $(\mathbf{S1})$ [3].

Theorem 1 (Network Decomposition): For an arbitrary (m, n) model, the following network decompositions hold:

$$\begin{aligned}
&(m, n) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2)\\
&= \begin{cases} (r, r+1)^{n-m-a} \times (r+1, r+2)^a, & m < n;\\ (r+1, r)^{m-n-a} \times (r+2, r+1)^a, & m > n. \end{cases}
\end{aligned}$$

where

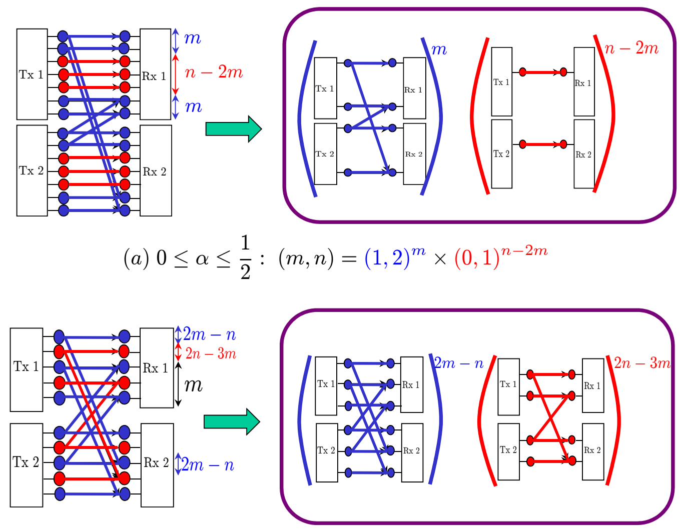
$$\begin{aligned}
r &= \left\lfloor \frac{\min\{m, n\}}{|n-m|} \right\rfloor,\\
a &= \min\{m, n\} \mod |n-m|.
\end{aligned} \qquad (3)$$

Here we use the symbol $\times$ for the concatenation of orthogonal models, just like in $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$.

Proof: See [3] for the complete proof. Here we only provide a network decomposition idea with two representative examples, each belonging to the regimes $0 \leq \alpha \leq \frac{1}{2}$ and $\frac{1}{2} \leq \alpha \leq \frac{2}{3}$ respectively. Here we define $\alpha := \frac{m}{n}$. Notice from (2) that

$$\begin{aligned}
(m, n) &= (0, 1)^{n-2m} \times (1, 2)^m, && 0 \leq \alpha \leq \frac{1}{2};\\
(m, n) &= (1, 2)^{2n-3m} \times (2, 3)^{2m-n}, && \frac{1}{2} \leq \alpha \leq \frac{2}{3}.
\end{aligned}$$

Fig. $2(a)$ shows the network decomposition for the regime of $0 \leq \alpha \leq \frac{1}{2}$. The decomposition idea is to use graph



(a) $0 \leq \alpha \leq \frac{1}{2}$: $(m, n) = (1, 2)^m \times (0, 1)^{n-2m}$

(b) $\frac{1}{2} \leq \alpha \leq \frac{2}{3}$: $(m, n) = (2, 3)^{2m-n} \times (1, 2)^{2n-3m}$

Fig. 2. Network Decomposition.

coloring. Start with assigning a color (say, blue) to the first level at transmitter 1. We then assign the same blue color to all the levels that are connected with the first level at transmitter 1. These are the first level at receiver 1 and the second bottom level at receiver 2. Now do the same procedure starting from transmitter 2. Specifically, assign the blue color to the first level at transmitter 2, then assign the same color to all of the connected levels: the first level at receiver 2 and the second bottom level at receiver 1. We then obtain an independent graph of model $(1, 2)$ and are left with model $(m-1, n-2)$. For the remaining graph of model $(m-1, n-2)$, repeat the above procedure. We then obtain $(1, 2)^2$ and are left with model $(m-2, n-2)$. Here we used the same blue color, as the additionally obtained graph is of the same model $(1, 2)$. Repeating this procedure m times, we finally obtain $(1, 2)^m \times (0, 1)^{n-m}$. Using the same graph coloring idea, we can also prove the second decomposition. See Fig. $2(b)$ for details. ■

Remark 1: Theorem 1 suggests that fundamental building blocks are of form $(r, r+1)$ or $(r+1, r)$, that is, "gap-1" models. □

IV. Modulo-2 Sum Multicast (S1)

In this scenario, both receivers want to decode the same function. Hence, the performance metric should be multicasting capacity defined as

$$\max_{(R_1, R_2) \in \mathcal{C}} \min\{R_1, R_2\}.$$

It can be readily seen that the multicast capacity is identical to the symmetric capacity defined in Section II. The symmetric capacity was already characterized both for the nonfeedback case [3] and for the feedback case [4]. Here we restate the main results in [3], [4] with an emphasis on the separation principle.

A. Nonfeedback Case

Theorem 2: The network decomposition (2) is optimal, i.e., coding separately over decomposed orthogonal subnetworks achieves the symmetric capacity:

$$C_{\mathsf{sym}} = \begin{cases} m, & 0 \le \alpha \le \frac{2}{3}; \\ \frac{2}{3}n, & \frac{2}{3} \le \alpha < 1; \\ n, & \alpha = 1; \\ \frac{2}{3}m, & 1 < \alpha \le \frac{3}{2}; \\ n, & \alpha \ge \frac{3}{2}, \end{cases}$$

where $\alpha := \frac{m}{n}$.

Proof: Since elementary subnetworks are of form $(r, r+1)$ or $(r+1, r)$ as suggested in Theorem 1, we focus on the rates of the "gap-1" models.

Lemma 1: The following rates are achievable:

(1) For the models of $(0,1)$ and $(1,0)$, $R_{\mathsf{sym}} = 0$.
(2) For the models of $(1,2)$ and $(2,1)$, $R_{\mathsf{sym}} = 1$.
(3) For the models of $(r, r+1)$ and $(r+1, r)$ with $r \ge 2$, $R_{\mathsf{sym}} = \frac{2}{3}(r+1)$.
(4) For the model (r,r), $R_{\mathsf{comp}} = r$.

The proof of this lemma is given in [3].

We will now show the optimality of the network decomposition. We focus on the case of $m < n$. The mirror case of $m > n$ similarly follows. The case of $m = n$ is straightforward. For the case of $0 \le \alpha \le \frac{1}{2}$, the decomposition is given by $(m,n) = (0,1)^{n-2m} \times (1,2)^m$. Thus, using Lemma 1, the symmetric rate is $R_{\mathsf{sym}} = 0 \cdot (n-2m) + 1 \cdot m = m$. For the case of $\frac{1}{2} \le \alpha \le \frac{2}{3}$, $(m,n) = (1,2)^{2n-3m} \times (2,3)^{2m-n}$. Thus, using Lemma 1, the symmetric rate is $R_{\mathsf{sym}} = 1 \cdot (2n-3m) + 2 \cdot (2m-n) = m$. Finally, consider the case of $\alpha \ge \frac{2}{3}$. Applying the decomposition (2), we find that in this case, $r \ge 2$. So we get

$$\begin{aligned} R_{\mathsf{comp}} &= \frac{2}{3}(r+1)(n-m-a) + \frac{2}{3}(r+2)a \\ &= \frac{2}{3}\left\{r(n-m) + a + (n-m)\right\} \\ &\overset{(a)}{=} \frac{2}{3}\left\{m + (n-m)\right\} = \frac{2}{3}n. \end{aligned}$$

where (a) is due to (3). Together with the converse proof established in [3], we complete the proof. ■

B. Feedback Case

Theorem 3: The network decomposition (2) is optimal, i.e., the separation approach can achieve the symmetric feedback capacity:

$$C_{\mathsf{sym}}^{\mathsf{FB}} = \begin{cases} \frac{2}{3}n, & \alpha < 1; \\ n, & \alpha = 1; \\ \frac{2}{3}m, & \alpha > 1. \end{cases}$$

Proof: We find that nontrivial "gap-1" models in the feedback case are of $(0,1)$, $(1,0)$, $(1,2)$ and $(2,1)$. We employ nonfeedback strategies for the other models. The symmetric feedback rates of the key models are given in the following lemma. See [4] for the proof.

Lemma 2: The following rates are achievable:

(1) For the models of $(0,1)$ and $(1,0)$, $R_{\mathsf{sym}}^{\mathsf{FB}} = \frac{2}{3}$.
(2) For the models of $(1,2)$ and $(2,1)$, $R_{\mathsf{sym}}^{\mathsf{FB}} = \frac{4}{3}$.

As in the nonfeedback case, we focus on the case of $m < n$. For the case of $0 \le \alpha \le \frac{1}{2}$, $(m,n) = (0,1)^{n-2m} \times (1,2)^m$. Using Lemma 2, we can then achieve $R_{\mathsf{sym}}^{\mathsf{FB}} = \frac{2}{3} \cdot (n - 2m) + \frac{4}{3} \cdot m = \frac{2}{3}n$. For the case of $\frac{1}{2} \le \alpha \le \frac{2}{3}$, $(m,n) = (1,2)^{2n-3m} \times (2,3)^{2m-n}$. Using Lemma 2, we get $R_{\mathsf{sym}}^{\mathsf{FB}} = \frac{4}{3} \cdot (2n-3m) + 2 \cdot (2m-n) = \frac{2}{3}n$. For the case of $\alpha \ge \frac{2}{3}$, we employ nonfeedback schemes to achieve $R_{\mathsf{sym}}^{\mathsf{FB}} = \frac{2}{3}n$. Using the converse proof established in [4], we complete the proof. ■

V. Two-Unicast (S2)

For simplicity, we examine the optimality of the separation approach only for the symmetric capacity, although the general performance metric is the capacity region in this scenario.

A. Nonfeedback Case

Theorem 4: The network decomposition (2) is optimal, i.e., coding separately over decomposed orthogonal subnetworks achieves the symmetric capacity [7], [8]:

$$C_{\mathsf{sym}} = \begin{cases} n-m, & 0 \le \alpha \le \frac{1}{2}; \\ m, & \frac{1}{2} \le \alpha \le \frac{2}{3}; \\ n - \frac{m}{2}, & \frac{2}{3} \le \alpha \le 1; \\ \frac{m}{2}, & 1 \le \alpha \le 2; \\ n, & \alpha \ge 2. \end{cases}$$

Proof: The symmetric rates of the gap-1 models are given in the following lemma. See [7], [8] for the proof.

Lemma 3: The following rates are achievable:

$(1a)$ For the model $(0,1)$, $R_{\mathsf{sym}} = 1$.
$(1b)$ For the model $(1,0)$, $R_{\mathsf{sym}} = 0$.
(2) For the model $(1,2)$, $R_{\mathsf{sym}} = 1$.
$(3a)$ For the model $(r, r+1)$ with $r \ge 2$, $R_{\mathsf{sym}} = \frac{r}{2} + 1$.
$(3b)$ For the model $(r+1, r)$ with $r \ge 1$, $R_{\mathsf{sym}} = \frac{r+1}{2}$.
(4) For the model (r,r), $R_{\mathsf{comp}} = \frac{r}{2}$.

For $0 \le \alpha \le \frac{1}{2}$, $(m,n) = (0,1)^{n-2m} \times (1,2)^m$. Hence using Lemma 3, we get $R_{\mathsf{sym}} = 1 \cdot (n-2m) + 1 \cdot m = n - m$. For $\frac{1}{2} \le \alpha \le \frac{2}{3}$, $(m,n) = (1,2)^{2n-3m} \times (2,3)^{2m-n}$. Using Lemma 3, we get $R_{\mathsf{sym}} = 1 \cdot (2n-3m) + 2 \cdot (2m-n) = m$. For $\frac{2}{3} \le \alpha \le 1$, we get

$$\begin{aligned} R_{\mathsf{sym}} &= \left(\frac{r}{2} + 1\right)(n-m-a) + \left(\frac{r+1}{2} + 1\right)a \\ &= \frac{(n-m)r + a}{2} + n - m \\ &\overset{(a)}{=} n - \frac{m}{2}. \end{aligned}$$

where (a) is due to (3). For $1 \le \alpha \le 2$, we get

$$\begin{aligned} R_{\mathsf{sym}} &= \left(\frac{r+1}{2}\right)(m-n-a) + \left(\frac{r+2}{2}\right)a \\ &= \frac{(m-n)r + a}{2} + \frac{m-n}{2} \\ &\overset{(a)}{=} \frac{m}{2}. \end{aligned}$$

where (a) is due to (3). Finally for $\alpha \geq 2$, the decomposition is given by $(m,n) = (1,0)^{m-2n} \times (2,1)^n$ and hence we get $R_{\mathsf{sym}} = n$. Using the converse proof established in [7], we complete the proof. ■

B. Feedback Case

Theorem 5: The network decomposition (2) is optimal, i.e., the separation approach can achieve the symmetric feedback capacity:

$$C_{\mathsf{sym}}^{\mathsf{FB}} = \begin{cases} n - \frac{m}{2}, & 0 \leq \alpha \leq 1; \\ \frac{m}{2}, & \alpha \geq 1. \end{cases}$$

Proof: It turns out that in the feedback case, nontrivial "gap-1" models are of $(1,0)$, $(1,2)$. We employ nonfeedback strategies for the other models. The symmetric feedback rates of the key models are given in the following lemma. See [9] for the proof.

Lemma 4: The following rates are achievable:

(1) For the model $(1,0)$, $R_{\mathsf{sym}}^{\mathsf{FB}} = \frac{1}{2}$.
(2) For the model $(1,2)$, $R_{\mathsf{sym}}^{\mathsf{FB}} = \frac{3}{2}$.

For the case of $0 \leq \alpha \leq \frac{1}{2}$, $(m,n) = (0,1)^{n-2m} \times (1,2)^m$. Using Lemma 4, we can then achieve $R_{\mathsf{sym}}^{\mathsf{FB}} = 1 \cdot (n-2m) + \frac{3}{2} \cdot m = n - \frac{m}{2}$. For $\frac{1}{2} \leq \alpha \leq \frac{2}{3}$, $(m,n) = (1,2)^{2n-3m} \times (2,3)^{2m-n}$. Using Lemma 4, we get $R_{\mathsf{sym}}^{\mathsf{FB}} = \frac{3}{2} \cdot (2n-3m) + 2 \cdot (2m-n) = n - \frac{m}{2}$. For the case of $\frac{2}{3} \leq \alpha \leq 2$, we employ nonfeedback schemes. Finally for $\alpha \geq 2$, $(m,n) = (1,0)^{m-2n} \times (2,1)^n$ and hence we get $R_{\mathsf{sym}}^{\mathsf{FB}} = \frac{1}{2} \cdot (m-2n) + 1 \cdot n = \frac{m}{2}$. Using the converse proof in [9], [10], we complete the proof. ■

VI. Classical Multicast (S3)

As in the first scenario (**S1**), the performance metric is the multicast capacity which coincides with the symmetric capacity defined in Section II. In this scenario, the symmetric capacity can be easily derived from the intersection of the capacity regions of two individual multiple access channels associated with two receivers. Specifically it is given by

$$\begin{aligned} C_{\mathsf{sym}} &= \sup\{R_1 + R_2 : (R_1, R_2) \in \mathcal{C}_{\mathsf{MAC1}} \cap \mathcal{C}_{\mathsf{MAC2}}\} \\ &= \min\{2\min(m,n), \max(m,n)\} \end{aligned}$$

where $\mathcal{C}_{\mathsf{MAC1}} = \{(R_1,R_2) : R_1 \leq n, R_2 \leq m, R_1 + R_2 \leq \max(m,n)\}$ and $\mathcal{C}_{\mathsf{MAC2}} = \{(R_1,R_2) : R_1 \leq m, R_2 \leq n, R_1 + R_2 \leq \max(m,n)\}$.

A. Nonfeedback Case

Theorem 6: The network decomposition (2) is optimal, i.e., coding separately over decomposed orthogonal subnetworks achieves the symmetric capacity:

$$C_{\mathsf{sym}} = \begin{cases} 2m, & 0 \leq \alpha \leq \frac{1}{2}; \\ n, & \frac{1}{2} \leq \alpha \leq 1; \\ m, & 1 \leq \alpha \leq 2; \\ 2n & \alpha \geq 2. \end{cases}$$

Proof: The symmetric rates of the gap-1 models are given in the following lemma. The proof is straightforward.

Lemma 5: The following rates are achievable:

(1) For the models of $(0,1)$ and $(1,0)$, $R_{\mathsf{sym}} = 0$.
(2) For the models of $(r, r+1)$ and $(r+1, r)$ with $r \geq 1$, $R_{\mathsf{sym}} = r+1$.
(3) For the model (r,r), $R_{\mathsf{comp}} = r$.

We focus on the case of $m \leq n$. The other case similarly follows. For $0 \leq \alpha \leq \frac{1}{2}$, $(m,n) = (0,1)^{n-2m} \times (1,2)^m$. Hence using Lemma 5, we get $R_{\mathsf{sym}} = 0 \cdot (n-2m) + 2 \cdot m = 2m$. For $\frac{1}{2} \leq \alpha \leq 1$, we get

$$\begin{aligned} R_{\mathsf{sym}} &= (r+1)(n-m-a) + (r+2)a \\ &= \{(n-m)r + a\} + n - m \\ &\overset{(a)}{=} n. \end{aligned}$$

where (a) is due to (3). This completes the proof. ■

B. Feedback Case

Theorem 7: The network decomposition (2) is optimal, i.e., the separation approach can achieve the symmetric feedback capacity [11]:

$$C_{\mathsf{sym}}^{\mathsf{FB}} = \begin{cases} n, & 0 \leq \alpha \leq 1; \\ m, \alpha & \geq 1. \end{cases}$$

Proof: It turns out that $(0,1)$ and $(1,0)$ are the only nontrivial models where feedback provides a capacity increase. The symmetric feedback rates of these models are given in the following lemma. See [11] for the proof.

Lemma 6: For the models of $(0,1)$ and $(1,0)$, $R_{\mathsf{sym}}^{\mathsf{FB}} = 1$.

By symmetry, it suffices to consider the case of $0 \leq \alpha \leq \frac{1}{2}$. In this case, $(m,n) = (0,1)^{n-2m} \times (1,2)^m$. The above lemma yields $R_{\mathsf{sym}}^{\mathsf{FB}} = 1 \cdot (n-2m) + 2 \cdot m = n$. This completes the proof together with the converse proof in [11]. ■

VII. Conclusion

We revisited the network composition for the two-transmitter two-receiver ADT network. We showed the optimality of our separation approach based on the network composition for three scenarios: (**S1**) modulo-2 sum multicast; (**S2**) two-unicast; (**S3**) classical multicast.

Our future work is along the following directions: (1) exploring the optimality of our separation approach for arbitrary ADT networks with four channel parameters; (2) extending to multi-hop ADT networks [12], [13]; (3) translating to the Gaussian wireless networks [6], [14].

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