Dynamic Subchannel and Bit Allocation for Multicast OFDM Systems

Changho Suh[†] and Chan-Soo Hwang[††]
[†]Samsung Electronics Co., Ltd, P.O.BOX 105, Suwon, S. Korea.
[††]i-Networking Lab., Samsung AIT, P.O.BOX 111, Suwon, S. Korea.
email: becal.suh@samsung.com, Tel: +82 31 279 5509, Fax: +82 31 279 5130

***Abstract*— In multicast orthogonal frequency division multiplexing (OFDM) systems, the difference in link conditions of users complicates adaptive modulation because modulation should be adjusted to serve the user who experiences the worst channel condition. If we assume that the multicast data are separated into layers and any combination of the layers can be decoded at the receiver, the network throughput can be increased by performing subcarrier/bit allocation. In this paper, in order to increase network throughput, we develop the optimum subcarrier/bit allocation method that maximizes the sum of data rate of all the users employing integer programming (IP) which is NP-hard problem. To reduce the complexity, suboptimum two-step algorithm is proposed: firstly, subcarriers are allocated to users under the assumption that the same power is distributed to each subcarrier; in the second step, the number of bits loaded to each subcarrier is determined using the modified Levin-Campello algorithm. Numerical results show that the performance difference between the optimum and suboptimum algorithms is within about 5%, and that total throughput of the proposed algorithm is larger than that of the lowest channel gain (LCG) method where modulation is determined to serve the user with the lowest channel gain.**

I. INTRODUCTION

Multicast delivers data to a group of users by a single transmission, which is particularly useful for high-data-rate multimedia service due to its ability to save the network resources. Since the bandwidth allocated to each user is different in heterogeneous network, the data rate of multicast stream is limited by the data rate of the least capable user, otherwise it is not delivered to multiple users. One approach of solving the heterogeneity is to exploit hierarchy in multicast data [1], [2], [3], [4]. For example, raw video data is compressed into a number of layers, arranged in a hierarchy that provides progressive refinement. If only the first layer is received by the user with the lowest data rate, the decoder produces the worst quality version. As more layers are received by more capable users, the decoder combines the layers to produce improved quality.

In wireless system, the spectrum is very scarce and the channel varies according to users due to Rayleigh fading; therefore, the multicast in wireless network should be spectrally efficient and be able to cope with the channel variation. The channel variation among users complicates adaptive modulation because the modulation should be adjusted to serve the user who experiences the worst channel condition; thus, it is usually adapted to the worst link condition [5], or the heterogeneity in link condition is often ignored during the design of adaptive modulation for multicast transmission [6]. To cope with the channel variation among users without adaptation, the non-uniform phase-shift-keying (PSK) is used in [3] where the base layer data is encoded to constellation points that are far apart in distance from each other than the higher layer data are encoded to. In [7], an adaptive modulation for multicast data is proposed assuming that the same modulation is used for all the subcarriers in an OFDM symbol.

In this paper, we propose a dynamic subcarrier/bit allocation method for multicast OFDM system assuming that the multicast data are separated into layers, and any combination of the layers can be decoded at the receiver. As a result, the data conveyed in OFDM subcarriers can be decoded by many users as long as the signal to noise ratio (SNR) of the user is larger than the minimum required SNR. Unlike the algorithm for subcarrier/bit allocation in multiuser OFDM systems [8], [9], we develop the optimum and suboptimum subcarrier/bit allocation method that maximizes the sum of data rate of all the users. Given the limited transmit power, the optimum algorithm for subcarrier/bit allocation is derived employing integer programming (IP) which is unfortunately an NP-hard problem. To reduce the complexity, suboptimum two-step algorithm is proposed: firstly, subcarrier allocation is performed under the assumption that the same power is distributed to each subcarrier; in the second step, the number of bits loaded to each subcarrier is determined using the modified Levin-Campello algorithm. Numerical results show that the performance difference between the optimum and suboptimum algorithms is within about 5%, and that total throughput of the proposed suboptimum algorithm is higher than that of the allocation scheme where modulation is determined to serve the user with the lowest channel gain.

II. PROBLEM FORMULATION

Multicast OFDM transmitter and receiver supporting K users are shown in Fig. 1. It is assumed that the multicast data are separated into layers, and any combination of the layers can be decoded at the receiver. The multicast data are fed into the adaptive modulator that assigns each subcarrier to a group of users who receive the same multicast data, and determines the number of bits on each subcarrier considering the lowest one among the channel gains of all users allocated to that subcarrier; therefore, channel information about all

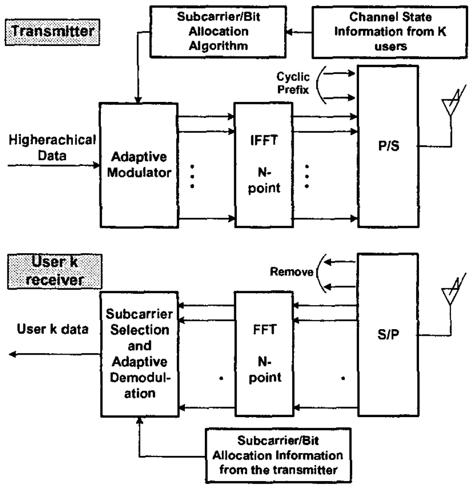


Fig. 1. Multicast OFDM Systems with Hierarchical Data

subcarriers of all users should be known to the transmitter, and the subcarrier/bit allocation information should be transmitted to each user through a separate control channel. Since the subcarrier/bit allocation information is available at the k-th user, subcarriers allocated to the user are selected and the signals associated with the subcarriers are demodulated, and then the signals are combined to reconstruct the original multicast data.

Assuming that the perceived quality of multicast data is proportional to the amount of data received by each user, the adaptive modulator in Fig. 1 should allocate subcarrier and load bits in a way that maximizes the total number of bits received by all the users. To describe the optimization procedure, we introduce notations that are adopted in [8], [9]. Let R_k be the data rate of the k-th user and c_n be the number of bits that are assigned to the n-th subcarrier. Here, the user index k is unnecessary because the users using the subcarrier receive the identical data using the same modulation. It is assumed that $c_n \in \mathbf{D} = \{0, 1, \cdots, M\}$ where M is the maximum number of bits/symbol that can be transmitted by each subcarrier. The data rate R_k can be expressed as

$$R_k = \sum_{n=1}^{N} c_n \rho_{k,n} \tag{1}$$

where $\rho_{k,n}$ is a binary value indicating whether the k-th user utilizes the n-th subcarrier or not.

$$\rho_{k,n} = \begin{cases} 1, & \text{if } n\text{-th subcarrier is used for } k\text{-th user} \\ 0. & \text{else} \end{cases} \tag{2}$$

The transmission power allocated to the n-th subcarrier is

$$P_n = \max_k P_{k,n} = \max_k \left(\frac{f(c_n)\rho_{k,n}}{\alpha_{k,n}^2} \right) \tag{3}$$

where $f(c_n)$ is the required receive power in the n-th subcarrier for reliable reception of c_n when the channel gain is unity. In practical system, if channel coding is considered in addition to adaptive modulation, $f(c_n)$ should be simply replaced by $g(c_n, r_n)$ which is calculated including code rate r_n. $\alpha_{k,n}^2$ indicates the channel gain of the n-th subcarrier of the k-th user. Since subcarrier can be shared by more than one user, maximum transmit power should be selected among the required transmit powers of selected users.

In multicast systems with hierarchical data, data rate is highly dependent on channel quality; hence, it is meaningful to solve the rate adaptive (RA) problem having the power constraint. In addition, since the fundamental problem of multicast system is that total throughput is reduced due to the dependency on the lowest channel gain, the optimization problem is formulated to maximize the total data rate of all the users without considering the fairness. Assuming that available total transmit power is limited by P_T, the optimization problem can be expressed as follows:

$$\begin{aligned} \max_{c_n, \rho_{k,n}} R_T &= \max_{c_n, \rho_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} c_n \rho_{k,n} \\ \text{subject to } &\sum_{n=1}^{N} \max_k \left(\frac{f(c_n)\rho_{k,n}}{\alpha_{k,n}^2} \right) \le P_T. \end{aligned} \tag{4}$$

This problem is nonlinear because of the nonlinearity of $f(c)$ and "max" function. For example, in the case of $M-$ary quadrature amplitude modulation (M-QAM), $f(c)$ can be represented as

$$f(c) = \frac{N_o}{3} \left[Q^{-1} (p_e/4) \right]^2 (2^c - 1) \tag{5}$$

where p_e is the required bit error rate (BER), $N_o/2$ denotes the variance of the additive white Gaussian noise (AWGN), and

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt \tag{6}$$

[8].

III. Optimum Algorithm - Integer Programming

In this section, the nonlinear optimization problem is converted into linear one by using the fact that c_n takes only integer values. In addition, the nonlinear constraint in Eq. (4) is converted into a number of linear constraints to obtain typical IP problem.

If M-QAM is used for the subcarrier, $f(c)$ are constants that can be calculated from Eq. (5) as follows:

$$f(c_n) = \{0, f(1), \cdots, f(M)\}. \tag{7}$$

In order to make $f(c_n)$ integer variable, the new indicator $\gamma_{k,n,c}$ is defined as follows:

$$\gamma_{k,n,c} = \begin{cases} \rho_{k,n}, & c_n = c \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

Once c_n is used for the n-th subcarrier, $\gamma_{k,n,c}$ should be zero for other values except c_n. This constraint can be expressed as

$$\begin{gathered}\left\{\begin{array}{l}1 \leq \sum_{k=1}^{K}\gamma_{k,n,1} \leq K \quad \text{and} \\ \sum_{c\neq 1}\sum_{k=1}^{K}\gamma_{k,n,c}=0\end{array}\right\} \text{or} \\ \vdots \\ \left\{\begin{array}{l}1 \leq \sum_{k=1}^{K}\gamma_{k,n,M} \leq K \quad \text{and} \\ \sum_{c\neq M}\sum_{k=1}^{K}\gamma_{k,n,c}=0\end{array}\right\}.\end{gathered} \tag{9}$$

Using $\gamma_{k,n,c}$ defined in Eq. (8), $c_n\rho_{k,n}$ and $f(c_n)\rho_{k,n}$ are given by respectively

$$\begin{aligned} c_n\rho_{k,n} &= \sum_{c=1}^{M} c\cdot\gamma_{k,n,c}, \\ f(c_n)\rho_{k,n} &= \sum_{c=1}^{M} f(c)\gamma_{k,n,c}. \end{aligned} \tag{10}$$

Finally, by replacing the constraint having "max" function by a set of linear equations, the above problem is converted into an optimization problem that can be solved by the IP having $\gamma_{k,n,c}$ as variable. Specifically, K^N linear constraints are introduced noting that the "max" function can be replaced by searching over all the possible choices of $\alpha_{k,n}$. With these constraints, the optimization problem in Eq. (4) can be converted into IP problem as follows:

$$\begin{aligned} &\max_{\gamma_{k,n,c}} R_T = \max_{\gamma_{k,n,c}} \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{c=1}^{M} c\cdot\gamma_{k,n,c} \\ &\text{subject to } \sum_{n=1}^{N}\sum_{c=1}^{M}\frac{f(c)\gamma_{1,n,c}}{\alpha_{1,n}^2} \leq P_T, \\ &\sum_{n=1}^{N-1}\sum_{c=1}^{M}\frac{f(c)\gamma_{1,n,c}}{\alpha_{1,n}^2} + \sum_{c=1}^{M}\frac{f(c)\gamma_{2,N,c}}{\alpha_{2,N}^2} \leq P_T, \\ &\vdots \\ &\sum_{n=1}^{N}\sum_{c=1}^{M}\frac{f(c)\gamma_{K,n,c}}{\alpha_{K,n}^2} \leq P_T, \end{aligned} \tag{11}$$

and the constraints in Eq. (9)

In general, IP problem is a kind of NP-hard one whose complexity increases exponentially with the number of constraints and variables. Since the number of constraints increases significantly with the number of users and subcarriers, the algorithm for IP problem is not practical for real-time implementation. In the following section, a suboptimum algorithm is described which consumes polynomial time.

IV. Suboptimum Two-Step Approach

In this section, we consider the suboptimum two-step approach to simplify the IP problem derived in Eq. (11). In the first step, the subcarriers are assigned under the assumption that transmit power of each subcarrier is constant, which is only used for subcarrier allocation. Next, bits are loaded to the subcarriers which are assigned in the first step.

A. Subcarrier Allocation

Provided that transmit power of each subcarrier is constant for all n, the optimization problem in Eq. (4) can be rewritten as

$$\begin{aligned} &\max_{c_n,\rho_{k,n}} R_T = \max_{c_n,\rho_{k,n}} \sum_{k=1}^{K}\sum_{n=1}^{N} c_n\rho_{k,n} \\ &\text{subject to } \max_k\left(\frac{f(c_n)\rho_{k,n}}{\alpha_{k,n}^2}\right) \leq \frac{P_T}{N} \quad \text{for all } n. \end{aligned} \tag{12}$$

In the constraint of Eq. (12), the number of bits to be received by each user can be calculated by tentatively assuming that the user utilizes the subcarrier. Specifically, let $c_{k,n}$ be the number of bits that can be received by the k-th user through the n-th subcarrier in the case of $\rho_{k,n}=1$. Then, the number of bits for the k-th user is given by

$$c_{k,n} = \min\left(f^{-1}\left(\frac{\alpha_{k,n}^2 P_T}{N}\right), M\right) \tag{13}$$

where $f^{-1}(\cdot)$ is the inverse function of $f(\cdot)$ defined in Eq. (5) and M is the largest modulation index. Since the function $f(\cdot)$ is monotonically increasing with c, the inverse function can be uniquely determined. In a way that maximizes total data rate, the subcarrier is allocated.

1) For the n-th subcarrier, calculate tentative total data rate $R_{T,k,n}^*$ when the k-th user is selected as the user requiring maximum power.

$$R_{T,k,n}^* = u_{k,n}c_{k,n} \tag{14}$$

where $u_{k,n}$ indicates the number of users who have channel gains larger than $\alpha_{k,n}^2$.

2) For the n-th subcarrier, select the user index κ_n maximizing $R_{T,k,n}^*$.

$$\kappa_n = \arg\max_k R_{T,k,n}^*. \tag{15}$$

Then, subcarrier allocation is completed as follows:

$$\rho_{k,n} = \left\{\begin{array}{ll} 1, & \alpha_{k,n}^2 \geq \alpha_{\kappa_n,n}^2 \\ 0, & \text{otherwise.}\end{array}\right. \tag{16}$$

B. Bit Loading

In this subsection, the bit loading algorithm is considered under the assumption that subcarrier allocation is completed. A kind of greedy algorithm called Levin-Campello algorithm in [8], [10] is modified to determine the number of bits loaded to each subcarrier. The Levin-Campello algorithm used in single user OFDM systems assigns bits to subcarrier one bit at a time, and in each assignment the subcarrier that requires the least incremental power is selected.

Let $\Delta P_n(c)$ denote the incremental power needed for transmitting one additional bit through the subcarrier n. When the number of bits loaded to the n-th subcarrier is c, $\Delta P_n(c)$ is given by

$$\Delta P_n(c) = \frac{f(c+1)-f(c)}{\alpha_{\kappa_n,n}^2 u_n} \tag{17}$$

where u_n is the number of users who share the n-th subcarrier, which is necessary because the incremental power is shared by the group of users allocated to the subcarrier.

$$u_n = \sum_{k=1}^{K} \rho_{k,n} \tag{18}$$

Using Eq. (17) and Eq. (18), the modified Levin-Campello algorithm is summarized as follows:

Initialization

Let $c_n = 0$ and evaluate $\Delta P_n(0)$ for all n

Let P_T^* be the tentative transmit power and $P_T^* = 0$

Bit LoadingIteration

repeat the following unless $P_T^* \geq P_T$

$n^* = \arg\min_n \Delta P_n(c_n)$

$P_T^* = P_T^* + \Delta P_{n^*}(c_{n^*})u_{n^*}$

$c_{n^*} = c_{n^*} + 1$

if $c_{n^*} = M$, set $\Delta P_{n^*}(c_{n^*}) = \infty$

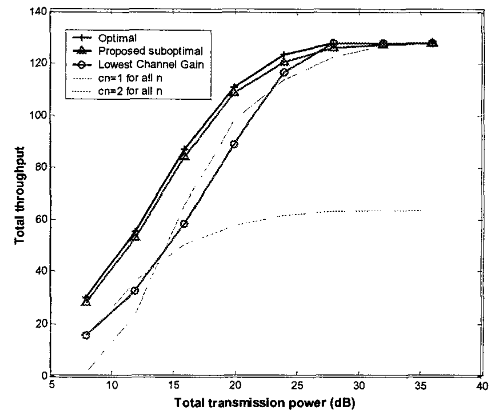
else evaluate $\Delta P_{n^*}(c_{n^*})$

In the above procedure, if c_{n^*} comes to M, ΔP_{n^*} should be set to the infinite value to prevent more bit loading. This algorithm provides the optimum bit loading solution [8], [10].

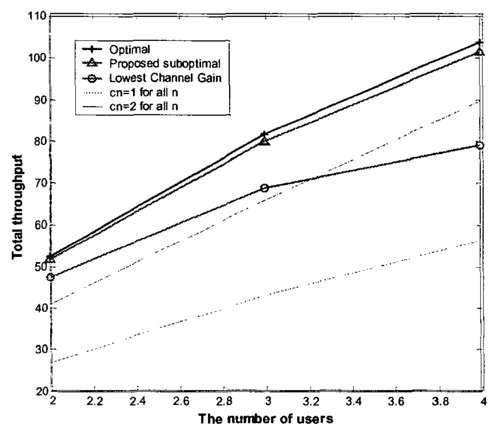
V. Numerical Results

In multicast OFDM systems, the proposed optimum and suboptimum algorithms are tested, and the throughput of the proposed suboptimum algorithm is compared with that of the lowest channel gain (LCG) method where all the subcarriers are shared by all the users and bits are loaded using the modified Levin-Campello algorithm. In the case of LCG method, only bit loading information is required at the receiver because all the users share the subcarrier. Simulations are performed under the following assumptions: the channel is a frequency selective Rayleigh fading channel with the equal gain profile; the required BER is $p_e = 10^{-4}$; the noise variance $N_o/2 = 1$; the number of users K is between two to 16. During the simulation, 100 independent channels are generated and the results in figures are the average of the total throughput of 100 trials.

Fig. 2 shows the comparison of IP optimum algorithm and suboptimum two-step algorithm when the number of subcarriers $N = 8$, maximum loaded bits $M = 2$, and the number of channel taps is four. In Fig. 2(a), the performance difference between the optimum and suboptimum algorithms is within about 5% for a wide range of transmission power, which indicates that the assumption that power is the same at all subcarriers is reasonable during subcarrier allocation. Compared with the LCG scheme and fixed modulation, e.g., $c_n = 1$ or $c_n = 2$, the performance difference between the optimum and suboptimum algorithms is insignificant. For large transmission power, it is observed that throughput is saturated regardless of algorithms because the maximum loaded bits are limited by two.



(a) Variation of total transmission power ($K = 4$)



(b) Variation of the number of users (P_T= 18 dB)

Fig. 2. Comparison of IP optimum algorithm and suboptimum two-step algorithm for $N = 8$ and $M = 2$

In Fig. 2(b), in order to evaluate total data rate for variable number of users, K is changed from two to four with the transmission power $P_T = 18$ dB. In the case of the optimum/suboptimum algorithms, total data rate increases with the number of users. While, in the case of LCG method, data rate becomes saturated because the probability that the value of the lowest channel gain becomes small is high for large number of users. Thus, it should be noted that the proposed scheme can be useful especially when there are many users. The comparison of optimum and suboptimum algorithms assumes small number of N, K and M because the number of constraints in IP problem exponentially increase with N, M, and K.

To show the performance gain of the proposed suboptimum algorithm in practical OFDM systems such as 802.11 [11], we consider the case of large parameters, e.g., $N = 64$, $M = 5$. In Fig. 3, total data rate with varying total transmission power

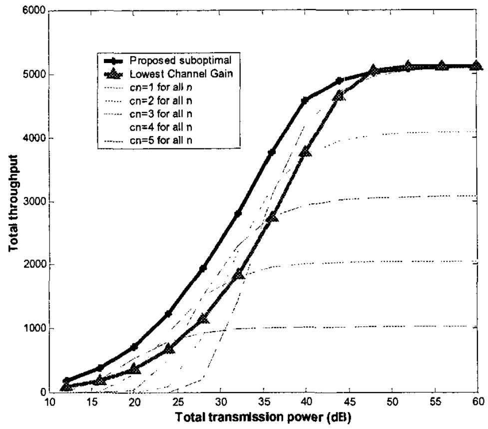


Fig. 3. Total throughput as a function of total transmission power P_T for $N = 64$, $K = 8$, and $M = 5$

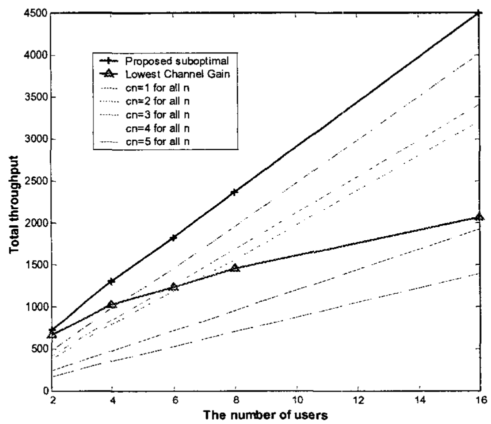


Fig. 4. Total throughput as a function of the number of users K for $N = 64$, $M = 5$, and $P_T = 30$ dB

is shown when $K = 8$ and the number of channel taps is eight. For a wide range of transmission power, the throughput of the proposed algorithm is always greater than the LCG scheme. For large transmission power, however, the LCG scheme is slightly better. Nevertheless, since we are interested in the case of insufficient transmission power in practical systems, the proposed suboptimum scheme is meaningful. In Fig. 4, we observe that the throughput of the proposed algorithm increases with the number of users because much more information can be shared with the increase of users. Finally, Fig. 5 indicates that data rate is almost independent from channel environment because the throughput is the same as varying the amount of frequency selectivity by increasing the number of taps from one to 16.

VI. Conclusions

In this paper, using the integer programming, the optimum algorithm for subcarrier/bit allocation has been developed in multicast OFDM systems under the assumption that hierarchical data can be combined using the data obtained from the algorithm. To reduce the complexity of IP algorithm, we also proposed the simplified suboptimum algorithm which has two steps: at fist, subcarrier allocation is executed assuming that each subcarrier has the same power; secondly, using the modified Levin-Campello algorithm, bits are loaded into subcarriers allocated in the first step. Through the simulations, it was shown that the performance difference between the optimum and suboptimum algorithms was within about 5%, and total throughput of simplified suboptimum algorithm was higher especially in the case of large number of users than that of the LCG method.

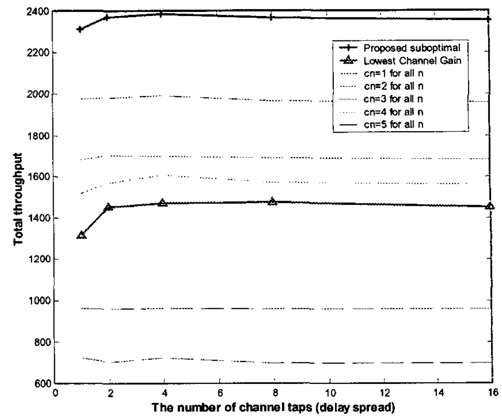


Fig. 5. Total throughput as a function of the maximum channel length for $N = 64$, $K = 8$, $M = 5$ and $P_T = 30$ dB

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