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# Dynamic Subchannel and Bit Allocation in Multiuser OFDM with a Priority User

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*Abstract*— **In this paper, we consider multiuser orthogonal frequency division multiplexing (OFDM) with adaptive subcarrier allocation and adaptive modulation, especially when there is one priority user who must be provided with a fixed data rate. We develop the optimum subcarrier/bit allocation method that minimizes total transmission power employing integer programming (IP) which is NP-hard problem. To reduce the complexity, suboptimum two-step algorithm is proposed: firstly, subcarriers are allocated to the priority user and then the remaining subcarriers are distributed to other users considering the best channel gain for each subcarrier; in the second step, using the Levin-Campello algorithm, the bits are loaded into the priority user and the other users separately. Numerical results show that total transmission power of the proposed optimum/suboptimum algorithms is significantly smaller than that of fixed modulation. In addition, the difference of total transmission power between the optimum and suboptimum algorithms is within about 0.5 dB when the number of subcarriers is 64 and the required data rate of the priority user is identical to the average required data rate of each user.**

## I. INTRODUCTION

High speed wireless data transmission requires robust and spectrally efficient modulation techniques. Orthogonal frequency division multiplexing (OFDM) systems have been applied to wireless communications, such as wireless local area network (LAN) [1], due to their robustness to multipath fading and high bandwidth efficiency. Since OFDM systems transmit the data through orthogonal subcarriers, modulation type and transmission power of each subcarrier can be different from those of other subcarriers. Thus, the spectral efficiency can be easily increased by allocating supportable bits and corresponding power to each subcarrier under the assumption that the channel state information (CSI) of each subcarrier is known to the transmitter.

In a single user OFDM system, the data rate can be increased or transmission power can be minimized using the CSI of subcarriers [2], [3]. In [2], the optimum solution, often called water-filling solution, was provided for maximizing the data rate given the constraint of total transmission power or minimizing transmission power given the constraint of data rate; however, it cannot be applied to the practical systems which have a finite granularity determined by the type of modulation and coding. In [3], bit loading technique, referred to as the Levin-Campello algorithm, was proposed to obtain the optimum solution in practical OFDM system.

In a multiuser OFDM system, the spectral efficiency can be more increased. For a ceratin subcarrier, the probability that all the users simultaneously have deep fading is very low. We assign the subcarrier to the user whose SNR is the best at that subcarrier and thus a sort of selection diversity can be achieved. It is often called multiuser diversity [4], [5], [6], [7]. Joint subcarrier/bit allocation algorithm has been dealt with in [5], where the solution was provided to minimize total transmission power under the constraint of the fixed data rate of each user. As a result, it can support the users whose data type is a kind of real-time multimedia. In [6], the optimum solution was obtained by converting the nonlinear optimization problem into linear one and it was solved by integer programming (IP). At the same time, to reduce the complexity of IP, the linear programming (LP) was introduced under the assumption that the constant bits are loaded to subcarriers allocated to each user. In [7], the solution was provided to maximize total data rate given the limited power; however, it cannot support the users requiring a fixed data rate, such as the users provided with real-time service. In practical systems such as [8], the user requiring a fixed data rate to support services like VOD/AOD, can coexist with the users who do not need the strict constraint of data rate. But, the above previous works cannot be applied to this case.

In this paper, we consider multiuser orthogonal frequency division multiplexing (OFDM) with adaptive subcarrier allocation and adaptive modulation when one user requiring a fixed data rate coexists with the users who do not need the strict constraint of data rate. It is assumed that the user who needs a fixed data rate is called the priority user in this paper. We develop the optimum subcarrier/bit allocation method that minimizes total transmission power by employing integer programming (IP) which is NP-hard problem. To reduce the complexity, suboptimum two-step algorithm is proposed: firstly, subcarriers are allocated to the priority user and then the remaining subcarriers are distributed to other users considering the best channel gain for each subcarrier; in the second step, using the Levin-Campello algorithm, the bits are loaded into the priority user and the other users separately. Numerical results show that total transmission power of the proposed optimum/suboptimum algorithms is significantly smaller than that of the fixed modulation. In addition, the total transmission power difference between the optimum and suboptimum algorithms is within about 0.5 dB when the number of subcarriers is 64 and the required data rate of the priority user is identical to the average required data rate of each user.

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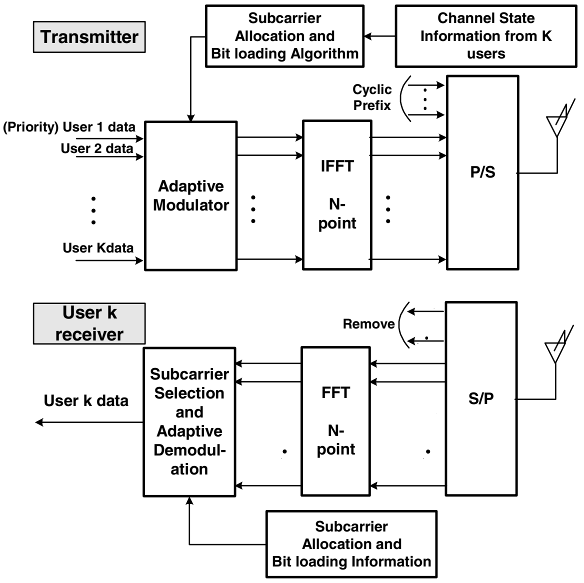

Fig. 1. Multiuser OFDM Systems with a Priority User

## II. SYSTEM MODEL

OFDM transmitter and receiver having $K$ users are shown in Fig. 1. To perform the subcarrier/bit allocation for all the users, channel state information (CSI) for all the subcarriers of all the users should be known to the transmitter, and the subcarrier and bit allocation information should be transmitted to each user through a separate control channel. We assume that each subcarrier should not be shared by different users. The frequency-domain symbols from the adaptive modulator are converted into time-domain by $N$-point inverse fast Fourier transform (IFFT). In our scenario, the bandwidth of each subcarrier is much smaller than the coherence bandwidth and the length of the cyclic prefix is longer than maximum channel length. The time-domain signal is transmitted from the antenna, and undergoes the frequency-selective Rayleigh-fading channel. The $k$-th receiver converts the received time-domain signal into the frequency-domain symbols using FFT after removing the cyclic prefix. Since the subcarrier and bit allocation information is available at the $k$-th user, subcarriers allocated to the user are selected and the signals associated with the subcarriers are demodulated.

The adaptive modulator in Fig. 1 allocates subcarrier and load bits in a way that minimizes total transmission power under the constraint that one priority user must be provided with a fixed data rate. For the other users, there is only one constraint that total data rate should be satisfied. To describe the optimization procedure, we introduce notations that are adopted in [5], [6]. Let $R_k$ be the data rate of the $k$-th user and $c_{k,n}$ be the number of bits of the $k$-th user that are assigned to the $n$-th subcarrier. Especially, let $R_1$ be the data rate of the priority user. It is assumed that $c_{k,n} \in \mathbf{D} = \{0, 1, \cdots, M\}$ where $M$ is the maximum number of bits/symbol that can be transmitted by each subcarrier. The data rate $R_k$ can be expressed as in [5], i.e.

$$R_k = \sum_{n=1}^{N} c_{k,n}\rho_{k,n} \tag{1}$$

where $\rho_{k,n}$ is a binary value indicating whether the $k$-th user occupies the $n$-th subcarrier or not.

$$\rho_{k,n} = \begin{cases} 1, & \text{if } n\text{-th subcarrier is used for } k\text{-th user} \\ 0, & \text{else} \end{cases} \tag{2}$$

Since we assume that sharing a subcarrier by different users is not allowed, $\rho_{k,n}$ should satisfy the following condition:

$$0 \leq \sum_{k=1}^{K} \rho_{k,n} \leq 1. \tag{3}$$

The transmission power allocated to the $n$-th subcarrier of the $k$-th user can be expressed as in [5], i.e,

$$P_{k,n} = \frac{f(c_{k,n})\rho_{k,n}}{\alpha_{k,n}^2} \tag{4}$$

where $f(c_{k,n})$ is the required receive power in the $n$-th subcarrier for reliable reception of $c_{k,n}$ when the channel gain is unity. $\alpha_{k,n}^2$ indicates the channel gain of the $k$-th user's $n$-th subcarrier.

Since we assume that the fixed data rate should be provided to the priority user and the total data rate of the other users are constant, the optimization problem to minimize total transmission power can be expressed as follows:

$$\begin{aligned} \min_{c_{k,n},\rho_{k,n}} P_T &= \min_{c_{k,n},\rho_{k,n}} \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{f(c_{k,n})}{\alpha_{k,n}^2} \cdot \rho_{k,n} \\ \text{subject to } &\sum_{k=2}^{K}\sum_{n=1}^{N} c_{k,n}\rho_{k,n} = R_T - R_1, \\ \sum_{n=1}^{N} c_{1,n}\rho_{1,n} &= R_1,\ 0 \leq \sum_{k=1}^{K}\rho_{k,n} \leq 1. \end{aligned} \tag{5}$$

This problem is nonlinear because $f(c)$ is nonlinear. For example, in the case of $M-$ary quadrature amplitude modulation (M-QAM), $f(c)$ can be represented as in [5], i.e.

$$f(c) = \frac{N_o}{3}\left[Q^{-1}\left(p_e/4\right)\right]^2 \left(2^c - 1\right) \tag{6}$$

where $p_e$ is the required bit error rate (BER), $N_o/2$ denotes the variance of the additive white Gaussian noise (AWGN), and

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt \tag{7}$$

[5].

## III. OPTIMUM ALGORITHM - INTEGER PROGRAMMING

In this section, the nonlinear optimization problem is converted into linear problem by using the fact that $c_{k,n}$ takes only integer values. Accordingly, typical IP problem can be obtained.

Provided that $c_{k,n} \in \{0, 1, \cdots, M\}$, then

$$f(c_{k,n}) = \{0, f(1), \cdots, f(M)\} \tag{8}$$

where $f(c)$ are constants that can be calculated from Eq. (6) when M-QAM is used for the subcarrier. In order to make $f(c_{k,n})$ integer variable, the indicator $\gamma_{k,n,c}$ can be defined as in [6], i.e.

$$\gamma_{k,n,c} = \begin{cases} 1, & c_{k,n} = c \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

for all $c \in \{0, 1, \cdots, M\}$.

Using $\gamma_{k,n,c}$ defined in (9), $c_{k,n}\rho_{k,n}$ and $f(c_{k,n})\rho_{k,n}$ are given by respectively,

$$\begin{aligned} c_{k,n}\rho_{k,n} &= \sum_{c=1}^{M} c \cdot \gamma_{k,n,c}, \\ f(c_{k,n})\rho_{k,n} &= \sum_{c=1}^{M} f(c)\gamma_{k,n,c}. \end{aligned} \tag{10}$$

From Eq. (10), the optimization problem in (5) can be converted into the IP problem as follows:

$$\begin{aligned} &\min_{\gamma_{k,n,c}} P_T = \min_{\gamma_{k,n,c}} \sum_{c=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N} \frac{f_k(c_{k,n})}{\alpha_{k,n}^2} \cdot \gamma_{k,n,c} \\ &\text{subject to } \sum_{c=1}^{M}\sum_{k=2}^{K}\sum_{n=1}^{N} c \cdot \gamma_{k,n,c} = R_T - R_1, \\ &\sum_{c=1}^{M}\sum_{n=1}^{N} \gamma_{1,n,c} = R_1,\ 0 \le \sum_{c=1}^{M}\sum_{k=1}^{K} \gamma_{k,n,c} \le 1. \end{aligned} \tag{11}$$

In general, IP problem is a kind of NP-hard one whose complexity increases exponentially with the number of constraints and variables. Thus, the algorithm for IP problem is not suitable to be used in the practical systems which require real-time implementation. In the following section, to reduce the complexity, a suboptimum, polynomial time algorithm is described.

## IV. SUBOPTIMUM TWO-STEP APPROACH

In this section, we consider the suboptimum two-step approach to simplify the IP problem derived in (11). In the first step, the subcarriers are allocated to the priority user and then the remaining subcarries are assigned to the other users with the following rule and assumption: the rule says that each subcarrier is allocated to the user with the best channel gain in the sense of minimizing total transmission power; the assumption is that the number of subcarriers allocated to the priority user is directly proportional to a required data rate. This assumption is only used during the subcarrier allocation. Next, bits are allocated to the priority user and the other users separately according to the subcarrier allocation completed in the first step. The separation of subcarrier allocation and bit loading enables an suboptimum algorithm, which is significantly simpler to implement than the IP-based optimum solution.

### A. Subcarrier Allocation

Provided that the number of subcarriers allocated to the priority user is directly proportional to the required data rate, the number of subcarriers allocated to the priority user and the remaining users are given by respectively,

$$\begin{aligned} n_1 &= \frac{R_1}{R_T} \cdot N, \\ n_o &= N - n_1. \end{aligned} \tag{12}$$

If we do not have a priority user, but only the constraint that total data rate of all the users are constant, the policy for subcarrier allocation in [7] is the optimum solution. In the presence of a priority user, however, this scheme may cause the violation of the constraint regarding the priority user. In addition, the violation can easily happen if the channel gains of the priority user are relatively smaller than those of the other users. On the contrary, this assumption regarding $n_1$ can always satisfy the constraint for the priority user by forcing subcarriers to be allocated to the priority user. In addition, if the average channel gain of the priority user is almost the same as those of the other users, this assumption becomes reasonable, which will be evaluated numerically.

In the sense of minimizing total transmission power, the subcarriers are allocated as the following rule:

1) For the priority user (the first user), $n_1$ subcarriers are selected successively according to the order of the best channel gain. Let $S^*$ and $S_1$ be the tentative set of subcarriers and the set of subcarriers of the priority user, respectively. Initially, $S^* = \{1, 2, \cdots, N\}$ and $S_1 = \emptyset$. The following procedure is performed $n_1$ times.

$$\begin{aligned} &< n_1\ times\ iteration > \\ &n^* = \arg\max_{n \in S^*} \alpha_{1,n}^2 \\ &S^* = S^* - \{n^*\} \\ &S_1 = S_1 \cup \{n^*\} \end{aligned} \tag{13}$$

   where $n^*$ is the tentative subcarrier index having the best channel gain among the subcarrier indices in $S^*$.

2) For the other users, select the user index $\kappa_n$ with the maximum channel gain for each subcarrier which belongs to $S_o$. Here, $S_o$ indicates the set of subcarriers for the other users; accordingly, $S_o = S^*$.

$$\kappa_n = \arg\max_{2 \le k \le K} \alpha_{k,n}^2, \quad n \in S_o. \tag{14}$$

3) The subcarrier allocation is completed as follows:

$$\begin{aligned} k = 1 \rightarrow \rho_{1,n} &= \begin{cases} 1, & n \in S_1, \\ 0, & \text{otherwise}, \end{cases} \\ 2 \le k \le K \rightarrow \rho_{k,n} &= \begin{cases} 1, & k = \kappa_n,\ n \in S_o, \\ 0, & \text{otherwise}. \end{cases} \end{aligned} \tag{15}$$

### B. Bit Loading

In this subsection, the bit loading algorithm is considered under the assumption that subcarrier allocation is completed. A kind of greedy algorithm called Levin-Campello algorithm in [3], [5] is used to determine the number of bits loaded to each subcarrier. The Levin-Campello algorithm used in single user OFDM systems assigns bits to subcarrier one bit at a time, and in each assignment the subcarrier that requires the least additional power is selected.

Let $\Delta P_{k,n}(c)$ denote the additional power needed for transmitting one additional bit through the subcarrier $n$ of the $k$-th user. When the number of bits loaded to the subcarriers is $c$, $\Delta P_{k,n}(c)$ is given by

$$\Delta P_{k,n}(c) = \begin{cases} \frac{f(c+1)-f(c)}{\alpha_{1,n}^2}, & k=1,\ n\in S_1,\\ \frac{f(c+1)-f(c)}{\alpha_{\kappa_n,n}^2}, & 2\le k\le K,\ n\in S_o.\end{cases} \qquad (16)$$

Using Eq. (16), the Levin-Campello algorithm is performed separately according to the subcarrier allocation of the priority user and the other users as follows:

```
Initialization
    Let $c_{k,n} = 0$ for all $k$ and $n$
    Evaluate $\Delta P_{1,n}(0)$ for $n \in S_1$
        and $\Delta P_{\kappa_n,n}(0)$ for $n \in S_o$
Bit LoadingIteration
    i) $k = 1$, repeat the following until satisfying $R_1$
        $n^* = \arg\min_{n\ \in S_1} \Delta P_{1,n}(c_{1,n})$
        $c_{1,n^*} = c_{1,n^*} + 1$
        if $c_{1,n^*} = M$, set $\Delta P_{1,n^*}(c_{1,n^*}) = \infty$
        else evaluate $\Delta P_{1,n^*}(c_{1,n^*})$
    ii) $k \neq 1$, repeat the following until satisfying $R_T$
        $n^* = \arg\min_{n\ \in S_o} \Delta P_{\kappa_n,n}(c_{\kappa_n,n})$
        $c_{\kappa_{n^*},n^*} = c_{\kappa_{n^*},n^*} + 1$
        if $c_{\kappa_{n^*},n^*} = M$, set $\Delta P_{\kappa_{n^*},n^*}(c_{\kappa_{n^*},n^*}) = \infty$
        else evaluate $\Delta P_{\kappa_{n^*},n^*}(c_{\kappa_{n^*},n^*})$
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In the above procedure, if $c_{k,n^*}$ comes to $M$, $\Delta P_{k,n^*}$ should be set to the infinite value to prevent more bit loading. Given the subcarrier allocation, this algorithm provides the optimum bit loading solution [3], [5].

## V. NUMERICAL RESULTS

The proposed optimum/suboptimum algorithms are tested in a multiuser OFDM system and the total transmission power of the proposed algorithm is compared with that of fixed modulation under the following assumptions: the channel is a frequency selective Rayleigh fading channel with an exponential decaying delay profile; the required BER is $p_e = 10^{-4}$; the noise variance $N_o/2 = 1$; the number of subcarriers $N = 64$; the maximum number of loaded bits $M = 5$; the number of users $K$ is between two to 16. During the simulation, 100 independent channels are generated and the results in the figures are the average of the total throughput of 100 trials.

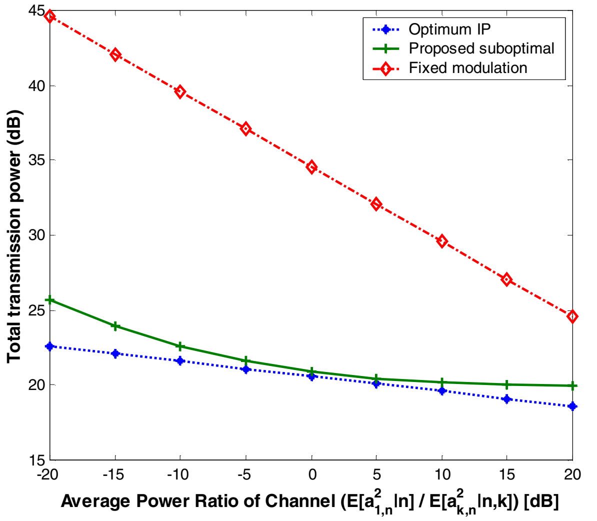

Fig. 2. Plots of total throughput as a function of the average power ratio of channel $E[\alpha_{1,n}^2|n]/E[\alpha_{k,n}^2|k,n]$, where $K = 4$, $R_1/R_T = 1/K$, $R_T = 256$, and $L = 8$

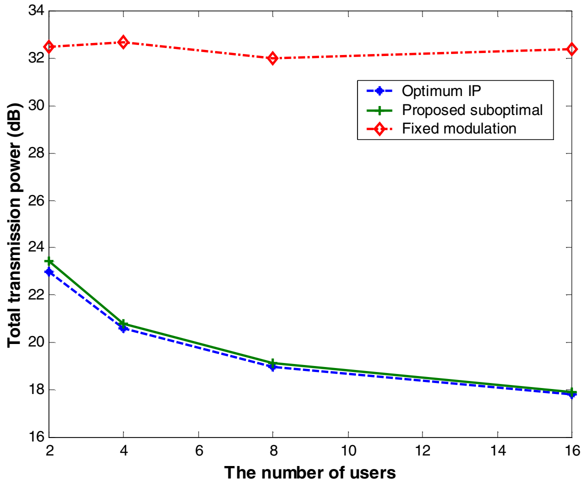

Fig. 3. Plots of total throughput as a function of the number of users for $R_1/R_T = 1/K$, $R_T = 256$, and $L = 8$

Fig. 2 show the comparison of IP optimum algorithm and suboptimum two-step algorithm as a function of the average channel power of the priority user to all the users, where we set $R_1/R_T = 1/K = 1/4$, and the number of channel taps $L = 8$. When the average power ratio is equal to one, i.e., 0 dB, the performance difference between the optimum and suboptimum algorithms is about 0.5 dB. Compared with the fixed modulation, we can say that the performance difference between the optimum and suboptimum algorithms is significantly small. Here, the fixed modulation follows the random loading where subcarriers are allocated randomly and constant bits are assigned without the knowledge of channel state information. As the average power ratio increases or decreases, the transmission power gap becomes broader. Accordingly, the

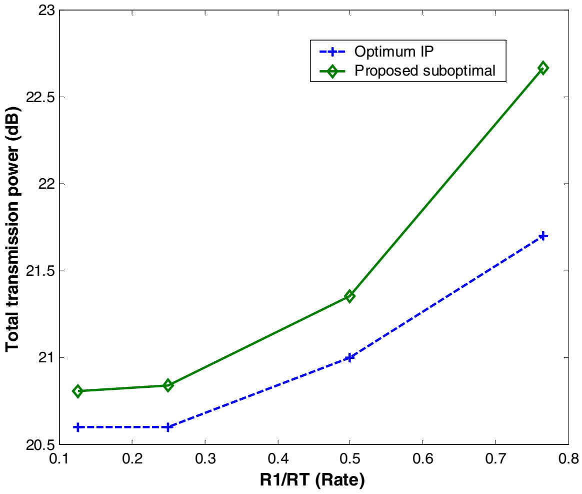

Fig. 4. Plots of total throughput as a function of the value of $R_1/R_T$ for $R_T = 256$, $K = 4$, and $L = 8$

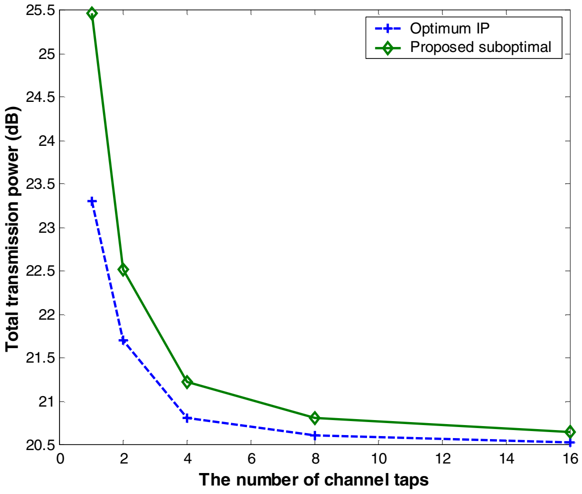

Fig. 5. Plots of total throughput as a function of the number of channel taps for $R_1/R_T = 1/K$, $K = 4$, and $R_T = 256$

assumption that the number of subcarriers allocated to the priority user is directly proportional to the required data rate is found to be reasonable if the average channel gain of the priority user is almost the same as those of the other users.

Fig. 3 shows the total transmission power as a function of the number of users K from two to 16, for $R_1/R_T = 1/K$, and the number of channel taps $L = 8$. For all the values of $K$, the difference of total transmission power between the optimum and suboptimum algorithms is within about 0.5 dB. Considering that the total transmission power decreases with the increase of the users, we confirm the effect of multiuser diversity.

In Fig. 4, the total transmission power with varying $R_1/R_T$ is shown for $K = 4$, $L = 8$, and $R_T = 256$. As $R_1$ increases, the total transmission power of the optimum/suboptimum algorithm will be increased significantly. At the same time, the transmission power difference between the optimum and suboptimum algorithms becomes larger as $R_1$ increases. In the case of $R_1/R_T = 0.25$ $(1/K)$, the transmission power gap is about 0.3 dB, while it is increased into 1.0 dB when $R_1/R_T$ equals to $0.75$. For large $R_1$, many subcarriers should be allocated to the priority user; hence, we can easily expect that the probability of using the subcarriers with the large channel gains will decrease.

In Fig. 5, in order to evaluate total transmission power for variable number of channel taps, $L$ is changed from one to 16 with the condition of $K = 4$, $R_1/R_T = 1/K$, and $R_T = 256$. The graph shows that transmission power decreases with the number of channel taps, that is, as the frequency selectivity increases, the effect of multiuser diversity is increased. In addition, the performance gap between the optimum/suboptimum algorithms becomes smaller as the frequency selectivity becomes higher.

## VI. CONCLUSIONS

In this paper, using the integer programming, the optimum subcarrier/bit allocation algorithm was developed in multiuser OFDM with a priority user. To reduce the complexity, we also proposed a two-step suboptimum algorithm. At first, assuming that the number of subcarriers allocated to the priority user is proportional to that of the required data rate, subcarriers are allocated to the priority user and remaining users. In the second step, employing the Levin-Campello algorithm, bits are loaded into the priority user and other users separately. Through the simulations, we have shown not only that total transmission power of the proposed optimum/suboptimum algorithms is significantly smaller than that of fixed modulation, but also that the transmission power difference between the optimum/suboptimum algorithms is within about 0.5 dB when $N = 64$ and $R_1/R_T = 1/K$. In addition, the performance gap decreases when the average power ratio of channel approaches to one, $R_1/R_T$ becomes small, or the frequency selectivity of channel becomes high.

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