

Resource Allocation for Multicast Services in Multicarrier Wireless Communications

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Abstract— In multicast wireless communications, the difference in link conditions of users due to fading limits the capacity of multicast data. In this paper, we confirm this by proving that in wireless fading channel, multicast capacity is saturated as the number of users increases. However, if we assume that the multicast data are separated into layers and any combination of the layers can be decoded at the receiver, the network throughput can be increased by performing subcarrier/bit allocation. In this paper, we develop the optimum subcarrier/bit allocation methods for not only maximum throughput (MT) but also proportional fairness (PF) by employing integer programming (IP) which is NP-hard problem. To reduce the complexity, suboptimum two-step algorithms are also proposed separating subcarrier allocation and bit loading. Numerical results show that the proposed resource allocation schemes for both MT and PF significantly outperform the conventional multicast transmission technique depending on the lowest channel gain. Additionally, it is shown that the performance difference between the optimum and suboptimum algorithms for both MT and PF is within about 5%.

I. INTRODUCTION

Multicast services have positive and negative sides for both wired and wireless communications. In wired communications where the link is relatively stable, the channel fluctuation across users is small; thus it simplifies the adaptive modulation for multicast data. However, in a single-link case,¹ a different wired link should be connected between the source and different destinations although the source delivers the same multicast data, i.e., as the number of multicast users increases, more wired links are required.

On the other hand, in wireless communications, multicast data can be delivered to many users only through a *single* transmission without increasing any wired connections. It is an attractive merit of multicasting in wireless channel. However, the enemy harassing multicast transmission in wireless communications is the difference of link conditions of users. In fact, the multicast capacity supporting all the users is saturated as the number of users goes to infinity, which will be briefly proved in the next section (See Theorem 1). One approach

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¹In a multi-link case, wire connections could be saved although the number of users increases. For this, however, we require intelligent information flow algorithms such as routing algorithms (not allowing coding operation at network node) and network coding schemes (allowing coding operation) [1].

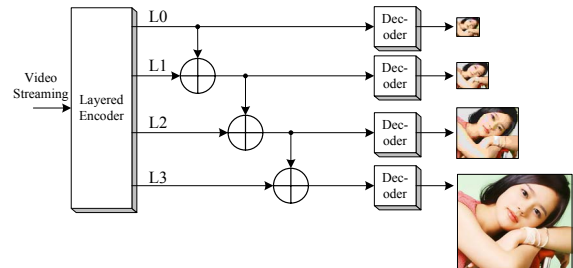


Fig. 1. Example of layered multicast data

of overcoming this capacity limitation is to exploit hierarchy in multicast data, which can be made by employing multi-resolution coding [2], [3], [4], [5]. Fig. 1 shows the example of layered multicast data. In this figure, raw video data is compressed into a number of layers, arranged in a hierarchy that provides progressive refinement. If only the first layer is received by the user with the lowest data rate, the decoder produces the worst quality version. As more layers are received by more capable users, the decoder combines the layers to produce improved quality.

In an attempt to increase network throughput in multicast systems, several techniques have been developed. To cope with the channel variation across users without adaptation, the non-uniform phase-shift-keying (PSK) is used in [4] where the base layer data is encoded to constellation points that are far apart in distance from each other than the higher layer data are encoded to. In [6], an adaptive modulation for multicast data was proposed assuming that the same modulation is used for all the subcarriers.

In this paper, we briefly investigate the capacity limitation of multicast data in wireless fading channel to recall a significant problem of conventional multicast transmission. As a way of overcoming this capacity limitation, we propose dynamic subcarrier/bit allocation method for maximum throughput (MT) assuming that the multicast data are separated into layers, and any combination of the layers can be decoded at the receiver

². In addition, to enforce the fairness performance while minimizing throughput degradation, we also propose subcarrier/bit allocation scheme for proportional fairness (PF). Given the limited transmit power, the optimum subcarrier/bit allocation algorithms for MT and PF are derived employing integer programming (IP) which is unfortunately an NP-hard problem. To reduce the complexity, suboptimum two-step algorithms are also proposed: firstly, subcarrier allocation is performed under the assumption that the same power is distributed to each subcarrier; in the second step, the number of bits loaded to each subcarrier is determined using the modified Levin-Campello algorithm. Numerical results show that the proposed resource allocation schemes for both MT and PF significantly outperform the conventional multicast transmission technique depending on the lowest channel gain. Additionally, it is shown that the performance difference between the optimum and suboptimum algorithms for both MT and PF is within about 5%.

This paper is organized as follows. In Section II, we present a significant problem of conventional multicast data in wireless fading channel. Section III formulates the optimization problem for overcoming the limitation of multicast capacity. And then, we propose the optimum/suboptimum resource allocation for MT and PF in Section IV and Section V, respectively. Section VI evaluates the performance of the proposed algorithms by comparing the conventional multicast transmission technique depending on the lowest channel gain. Finally, conclusions and discussions will be followed in Section VIII and Section VII, respectively.

II. CAPACITY LIMITATION OF CONVENTIONAL MULTICAST DATA

In conventional multicast transmission, capacity is adjusted to the user who experiences the worst channel condition; hence, for multicast services, an ergodic capacity can be defined as follows.

Definition 1 (Ergodic Multicast Capacity): Since conventional multicast data is shared by all the users, capacity can be defined as

$$E[C_{MC}] \triangleq E \left[K \cdot \min_{1 \leq k \leq K} \log_2 \left(1 + \frac{P}{\sigma^2} X_k \right) \right], \quad (1)$$

where K is the number of users and X_k is a random variable indicating the channel gain of user k . ■

We assume that the channel gain X_k of user k is independent and identically distributed with exponential with parameter α_k^2 , i.e.,

$$X_k \sim \frac{1}{\alpha_k^2} e^{-\frac{x}{\alpha_k^2}}, \quad \forall k. \quad (2)$$

For convenience, let us define the first order statistic [7]:

$$Y_{(1)} = \min\{X_1, X_2, \dots, X_K\}. \quad (3)$$

²To make this assumption valid, we may require specific source coding schemes in which data can be recovered with some different layers although the basic ones are not supported. It could be a further work of this paper.

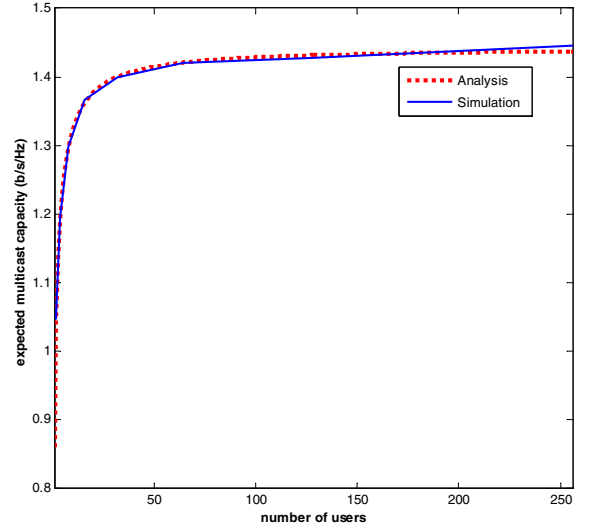


Fig. 2. Comparison of analysis and simulation results for expected multicast capacity ($P = 1$, $\sigma^2 = 1$, and $\alpha_{(1)}^2 = 1$)

Using the order statistics, we can easily obtain the following lemma.

Lemma 1:

$$\begin{aligned} E[C_{MC}] &= E \left[K \cdot \log_2 \left(1 + \frac{P}{\sigma^2} Y_1 \right) \right] \\ &= -\log_2 e \cdot K e^{\frac{K\sigma^2}{\alpha_{(1)}^2 P}} \text{Ei} \left(-\frac{K\sigma^2}{\alpha_{(1)}^2 P} \right), \end{aligned} \quad (4)$$

where $\text{Ei}(-x)$ is an exponential integral function defined in [8]: $\text{Ei}(-x) = -\int_x^\infty \frac{e^{-t}}{t} dt$ for $x > 0$.

Proof: See Appendix I. ■

From Lemma 1, we obtain the following theorem.

Theorem 1: For $K \rightarrow \infty$, we have

$$\lim_{K \rightarrow \infty} E[C_{MC}] = \log_2 e \cdot \frac{P\alpha_{(1)}^2}{\sigma^2} \quad (5)$$

Proof: See Appendix II. ■

Theorem 1 tells us that in wireless fading channel, multicast capacity is saturated as the number of users increases, i.e., the benefit of multicasting becomes perished in wireless fading channel. Fig. 2 numerically confirms Theorem 1. From the following section, we present the subcarrier/bit allocation scheme to overcome the capacity limitation of conventional multicast data.

III. PROBLEM FORMULATION

In order to increase network throughput, we consider hierarchical data having layered structure. Fig. 3 shows multicast multicarrier transmitter and receiver supporting K users. The original multicast data is encoded into hierarchical data having layered data structure. It is assumed that the hierarchical multicast data are separated into layers, and any combination of the layers can be decoded at the receiver. The hierarchical multicast data are fed into the subcarrier/bit allocator that assigns each subcarrier to a group of users who receive the

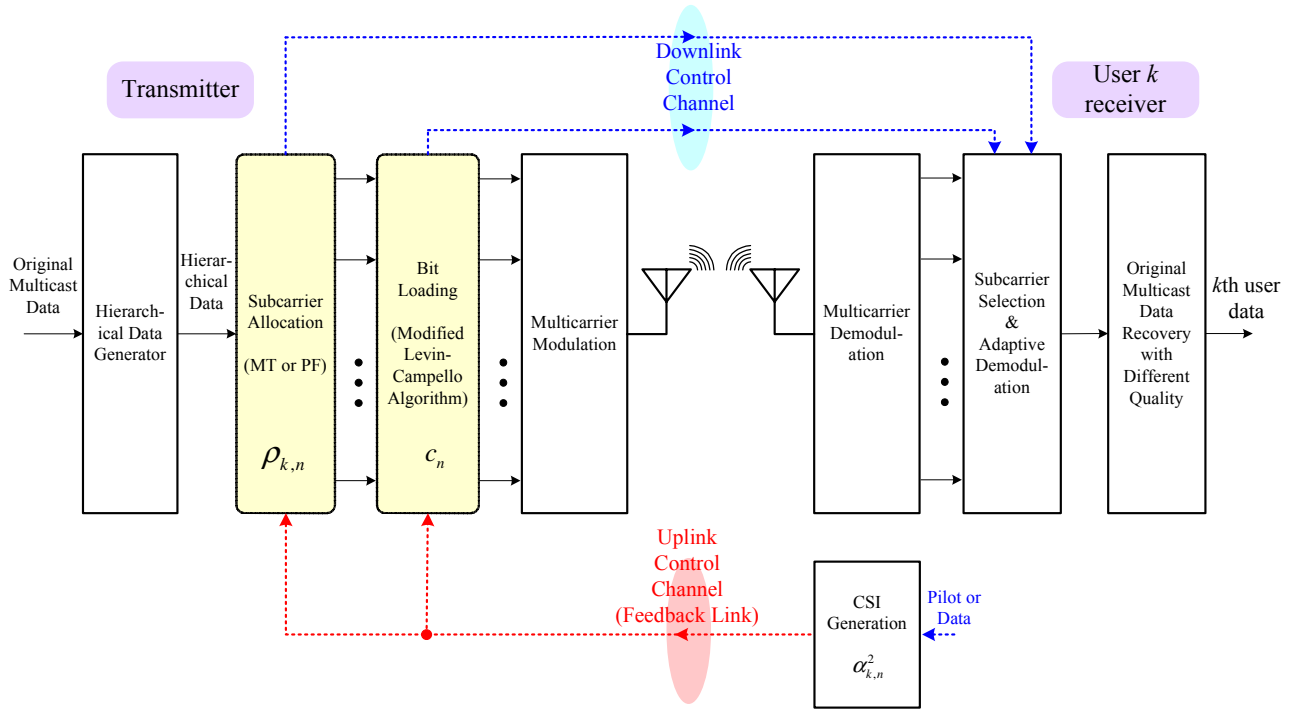


Fig. 3. Block diagram of multicast multicarrier systems with hierarchical data. The yellow blocks for subcarrier allocation and bit loading are main contributions of this paper. The blue and red dotted lines denote the downlink and uplink control channels, respectively.

same multicast data, and determines the number of bits on each subcarrier considering the lowest one among the channel gains of all users allocated to that subcarrier; therefore, channel state information (CSI) for all subcarriers of all users should be known to the transmitter, and the subcarrier/bit allocation information should be transmitted to each user through a separate control channel. Since the subcarrier/bit allocation information is available at the k th user, subcarriers allocated to the user are selected and the signals associated with the subcarriers are demodulated. Finally, the demodulated signals are combined to reconstruct the original multicast data.

If we assume that the perceived quality of multicast data is proportional to the amount of data received by each user, the subcarrier/bit allocator in Fig. 3 should allocate subcarrier and load bits in a way that maximizes total number of bits received by all the users. To describe the optimization procedure, we introduce notations that are adopted in [9], [10]. Let R_k be the data rate of the k th user and c_n be the number of bits that are assigned to the n th subcarrier. Here, the user index k is unnecessary because the users in the group receive the identical data using the same modulation. It is assumed that $c_n \in \mathbf{D} = \{0, 1, \dots, M\}$ where M is the maximum number of bits/symbol that can be transmitted by each subcarrier. The data rate R_k can be expressed as

$$R_k = \sum_{n=1}^N c_n \rho_{k,n} \quad (6)$$

where $\rho_{k,n}$ is a binary value indicating whether the k th user

utilizes the n th subcarrier or not.

$$\rho_{k,n} = \begin{cases} 1, & \text{if } n\text{th subcarrier is used for } k\text{th user} \\ 0. & \text{else} \end{cases} \quad (7)$$

The transmission power allocated to the n th subcarrier is

$$P_n = \max_k P_{k,n} = \max_k \left(\frac{f(c_n) \rho_{k,n}}{\alpha_{k,n}^2} \right) \quad (8)$$

where $f(c_n)$ is the required receive power in the n th subcarrier for reliable reception of c_n when the channel gain is unity. In practical system, if channel coding is considered in addition to adaptive modulation, $f(c_n)$ should be simply replaced by $g(c_n, r_n)$ that can be numerically or analytically calculated for code rate r_n . The parameter $\alpha_{k,n}^2$ indicates the channel gain of the n th subcarrier of the k th user. Since subcarrier can be shared by more than one user, maximum transmit power should be selected among the required transmit powers of selected users.

In multicast systems with hierarchical data, data rate is highly dependent on channel quality; hence, it is meaningful to solve the rate adaptive (RA) problem having the power constraint. As shown in Theorem 1, the fundamental problem of multicast system is that total throughput is saturated due to the dependency on the lowest channel gain. Thus, at first the optimization problem is set to maximize total data rate of all the users without considering the fairness. Assuming that available total transmit power is limited by P_T , the

optimization problem can be expressed as follows:

Maximum Throughput:

$$\begin{aligned} \max_{c_n, \rho_{k,n}} R_T &= \max_{c_n, \rho_{k,n}} \sum_{k=1}^K \sum_{n=1}^N c_n \rho_{k,n} \\ \text{subject to} \quad &\sum_{n=1}^N \max_k \left(\frac{f(c_n) \rho_{k,n}}{\alpha_{k,n}^2} \right) \leq P_T. \end{aligned} \quad (9)$$

This problem is nonlinear because of the nonlinearity of $f(c)$ and \max function. For example, in the case of M -ary quadrature amplitude modulation (M-QAM), $f(c)$ can be represented as

$$f(c) = \frac{N_o}{3} [Q^{-1}(p_e/4)]^2 (2^c - 1) \quad (10)$$

where p_e is the required bit error rate (BER), $N_o/2$ denotes the variance of the additive white Gaussian noise (AWGN), and

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt \quad (11)$$

[9].

On the other hand, in practical systems, we should consider not only throughput but also fairness. The proportional fairness (PF) scheduling can be a suitable policy because it tries to improve the fairness altogether with increasing the network throughput [11]. Although max-min fairness scheduling guarantees the fairness better than PF scheduling, it performs significantly worse from the perspective of throughput.³ Thus, it is not helpful for solving the fundamental problem of multicast systems that total throughput is critically reduced due to the dependency on the lowest channel gain.

The proportional fairness was firstly defined in [11] and later it was proved in [13], [14] that an allocation problem for proportional fairness is equivalent to the optimization problem for maximizing the sum of logarithmic data rate. In addition, for a low computational complexity, the authors in [15], [16] developed the simplified PF algorithm by employing the average data rate (ADR), which is given by

$$R_k(t) = \left(1 - \frac{1}{T_W}\right) R_k(t-1) + \frac{1}{T_W} \sum_{n=1}^N c_n \rho_{k,n}, \quad (12)$$

where T_W indicates the average window size. The value of $T_W = 1000$ was suggested for CDMA-HDR systems [16].

Taking all these into considerations, the optimization RA

³By the property that all data rates are almost identical in max-min fairness scheduling, it is often employed in wired communications including a lot of links where a bottleneck link significantly affects the system performance [12].

problem for PF scheduling can be written as

Proportional Fairness:

$$\begin{aligned} \max_{c_n, \rho_{k,n}} \sum_{k=1}^K \log R_k(t) &= \max_{c_n, \rho_{k,n}} \prod_{k=1}^K R_k(t) \\ \text{subject to given} \quad &R_k(t-1) \\ \text{and} \quad &\sum_{n=1}^N \max_k \left(\frac{f(c_n) \rho_{k,n}}{\alpha_{k,n}^2} \right) \leq P_T. \end{aligned} \quad (13)$$

IV. MAXIMUM THROUGHPUT

In this section, the optimum algorithm for maximum throughput (MT) is developed by using *integer programming* (IP). And then, to alleviate the burden of high computational complexity of IP problem, we propose the suboptimum two-step algorithm separating subcarrier allocation and bit loading.

A. Optimum Algorithm - Integer Programming

In order to obtain a typical integer programming (IP) problem, the nonlinear object function and constraints in the optimization problem should be converted into linear ones.⁴

To linearize the nonlinear function of $f(c)$, consider the fact that c_n takes only integer values. Then, for M-QAM constellation, $f(c)$ becomes constant as follows:

$$f(c_n) = \{0, f(1), \dots, f(M)\}, \quad (14)$$

which can be calculated from Eq. (10). In order to make $f(c_n)$ integer variable, the new indicator $\gamma_{k,n,c}$ is defined as follows:

$$\gamma_{k,n,c} = \begin{cases} \rho_{k,n}, & c_n = c, \\ 0, & \text{otherwise.} \end{cases} \quad (15)$$

Once c_n is used for the n th subcarrier, $\gamma_{k,n,c}$ should be zero for other values except c_n . Hence, a new constraint for $\gamma_{k,n,c}$ should be taken into account as

$$\begin{aligned} &\left\{ \begin{array}{l} 1 \leq \sum_k \gamma_{k,n,1} \leq K \quad \text{and} \\ \sum_{c \neq 1} \sum_k \gamma_{k,n,c} = 0 \end{array} \right\} \text{ or} \\ &\quad \vdots \\ &\left\{ \begin{array}{l} 1 \leq \sum_k \gamma_{k,n,M} \leq K \quad \text{and} \\ \sum_{c \neq M} \sum_k \gamma_{k,n,c} = 0 \end{array} \right\}. \end{aligned} \quad (16)$$

In addition, using $\gamma_{k,n,c}$ defined in Eq. (15), $c_n \rho_{k,n}$ and $f(c_n) \rho_{k,n}$ are given by respectively

$$\begin{aligned} c_n \rho_{k,n} &= \sum_{c=1}^M c \cdot \gamma_{k,n,c}, \\ f(c_n) \rho_{k,n} &= \sum_{c=1}^M f(c) \gamma_{k,n,c}. \end{aligned} \quad (17)$$

Finally, by replacing the constraint in Eq. (9) having \max function with a set of linear equations, the nonlinear problem is converted into a linear one that can be solved by the IP having $\gamma_{k,n,c}$ as variable. Specifically, K^N linear constraints

⁴In order to solve IP problem, both the object function and the constraints should be linear. This problem can be easily solved by using software tools such as CPLEX.

are introduced, which notes that the *max* function can be replaced by searching over all the possible choices of $\alpha_{k,n}$. With these constraints, the optimization problem in Eq. (9) can be converted into IP problem as follows:

$$\begin{aligned} \max_{\gamma_{k,n,c}} R_T &= \max_{\gamma_{k,n,c}} \sum_{k=1}^K \sum_{n=1}^N \sum_{c=1}^M c \cdot \gamma_{k,n,c} \\ \text{subject to} \quad &\sum_{n=1}^N \sum_{c=1}^M \frac{f(c)\gamma_{1,n,c}}{\alpha_{1,n}^2} \leq P_T, \\ &\sum_{n=1}^{N-1} \sum_{c=1}^M \frac{f(c)\gamma_{1,n,c}}{\alpha_{1,n}^2} + \sum_{c=1}^M \frac{f(c)\gamma_{2,N,c}}{\alpha_{2,N}^2} \leq P_T, \quad (18) \\ &\vdots \\ &\sum_{n=1}^N \sum_{c=1}^M \frac{f(c)\gamma_{K,n,c}}{\alpha_{K,n}^2} \leq P_T, \end{aligned}$$

and the constraints in Eq. (16)

In general, IP problem is a kind of NP-hard one whose complexity increases exponentially with the number of constraints and variables. In this case, since the number of constraints increases significantly with the number of users and subcarriers, the algorithm for IP problem is not suitable for being applied to practical systems requiring real-time implementation. In the following subsection, a suboptimum algorithm consuming polynomial time is developed.

B. Suboptimum Algorithm - Two Step Approach

In this subsection, we consider the suboptimum two-step approach to simplify the IP problem derived in Eq. (18). In the first step, the subcarriers are assigned under the assumption that transmit power of each subcarrier is constant, which is used only for subcarrier allocation. Next, bits are loaded to the subcarriers assigned in the first step. The separation of subcarrier allocation and bit loading enables an suboptimum algorithm; however, it makes the complexity significantly lower than the IP-based optimum solution.

Subcarrier Allocation: Assume that power allocation for each subcarrier is given by vector $P = (P_1, P_2, \dots, P_N)$. Then problem (9) is separable with respect to each subcarrier and the subcarrier n problem is:

$$\begin{aligned} \max_{c_n, \rho_{k,n}} c_n \sum_{k=1}^K \rho_{k,n} \\ \text{subject to} \quad \max_k \left(\frac{f(c_n)\rho_{k,n}}{\alpha_{k,n}^2} \right) \leq P_n \quad . \end{aligned} \quad (19)$$

Without loss of generality, we can assume that $\alpha_{k,n}$ is sorted in a decreasing order, i.e. $\alpha_{1,n} \geq \alpha_{2,n} \geq \dots \geq \alpha_{K,n}$. The channel quality the first user is the best while that of the last user is worst. Since $\rho_{k,n}$ is either 0 or 1, the term $\sum_{k=1}^K \rho_{k,n}$ represents the number of users that receive data for subcarrier n . Let $k_n^c(P)$ be the number of supportable users for a given

power P and c bits, i.e.

$$k_n^c(P_n) = \max \left\{ k \mid \alpha_{k,n}^2 \geq \frac{f(c)}{P_n} \right\}. \quad (20)$$

Then it is not difficult to check the feasibility of the following allocation,

$$\rho_{k,n} = \begin{cases} 1, & k \leq k_n^c(P_n) \\ 0, & \text{otherwise.} \end{cases} \quad (21)$$

from the way Eq. (20) is defined. The following theorem suggests another equivalent form of Problem (19).

Theorem 2: For a given P_n , Problem (19) is equivalent to

$$c_n^* = \arg \max_{c=1,2,\dots,M} c \cdot k_n^c(P_n), \quad (22)$$

where $k_n^c(P_n)$ is defined in Eq. (20) and P_n indicates the transmit power of subcarrier n .

Proof: Let $\rho_{k,n}^*$ be an optimal allocation. We first argue that $\rho_{k,n}^* = 1$ implies $\rho_{j,n}^* = 1$ for all $j \leq k$. Otherwise, there exists $j' (\leq k)$ such that $\rho_{j',n}^* = 0$ when $\rho_{k,n}^* = 1$. Since $\frac{f(c)}{\alpha_{j',n}^2} \leq \frac{f(c)}{\alpha_{k,n}^2}$, setting $\rho_{j',n}$ does not violate the constraint of (19), but the objective function value is higher, which contradicts the optimality of ρ^* . Since the objective function is increasing in $\sum_k \rho_{k,n}$, given c_n and P_n , the problem is equivalent to $\max_k \rho_{k,n}$ finding the marginal number of users which does not violate the constraint. From the first argument, the optimal ρ^* is equivalent to find the number of supported users. From the next argument, the number of user is limited by the constraint. Therefore, if we combine the two arguments, (22) is equivalent to the original problem. ■

Theorem 2 provides a polynomial time algorithm that requires only M evaluations of $c_n k_n^c$. Note that computation of k_n using (20) can be done in linear time. Therefore, the following algorithm is linear algorithm:

< MT Subcarrier Allocation >

- 1) Calculate $k_n^c(P)$ by (20) for all n and c .
- 2) Obtain c_n^* for all subcarriers $n = 1, \dots, N$ using (22).
- 3) Complete the subcarrier allocation using (21) with $k_n^{c_n^*}$.

It is possible to determine the optimal number of users in each subcarrier once subcarrier power allocation is given. Then, the next question is how to determine subcarrier power allocation. One of the simplest way is equal power allocation, i.e. $P = (\frac{P_T}{N}, \dots, \frac{P_T}{N})$. The next theorem gives a hint on how to determine subcarrier power allocation.

Bit Loading: Assume that the number of users k_n to support in each subcarrier n is given. We select them based on their channel quality $\alpha_{k,n}$ and those with larger $\alpha_{k,n}$ are selected. The problem (9) becomes

$$\begin{aligned} \max_{c_n} R_T &= \max_{c_n} \sum_{n=1}^N c_n \cdot k_n \\ \text{subject to} \quad &\sum_{n=1}^N \frac{f(c_n)}{\alpha_{k_n,n}^2} \leq P_T. \end{aligned} \quad (23)$$

The bit loading algorithm is considered under the assumption that subcarrier allocation is completed. A kind of greedy

algorithm called Levin-Campello algorithm in [9], [17] is modified to determine the number of bits loaded to each subcarrier. The Levin-Campello algorithm used in single user OFDM systems assigns bits to subcarrier *one bit at a time*, and in each assignment the subcarrier that requires the least additional power is selected.

Let $\Delta P_n(c)$ denote the additional power needed for transmitting one additional bit through the subcarrier n . When the number of bits loaded to the n th subcarrier is c , $\Delta P_n(c)$ is given by

$$\Delta P_n(c) = \frac{f(c+1) - f(c)}{\alpha_{\kappa_n, n}^2 k_n}, \quad (24)$$

where k_n is the number of users who share the n -th subcarrier. This factor is necessarily required because the incremental power is shared by the group of users who are allocated to the subcarrier. This is the main different point when compared to a well-known Levin-Campello algorithm. It is given by

$$k_n = \sum_{k=1}^K \rho_{k,n}. \quad (25)$$

Using Eq. (24) and Eq. (25), the modified Levin-Campello algorithm is summarized as follows:

<Modified Levin-Campello Algorithm >

Initialization

Let $c_n = 0$ and evaluate $\Delta P_n(0)$ for all n

Let P_T^* be the tentative transmit power and $P_T^* = 0$

Bit Loading Iteration

repeat the following unless $P_T^* \geq P_T$

$n^* = \arg \min_n \Delta P_n(c_n)$

$P_T^* = P_T^* + \Delta P_{n^*}(c_{n^*}) \cdot u_{n^*}$

$c_{n^*} = c_{n^*} + 1$

if $c_{n^*} = M$, set $\Delta P_{n^*}(c_{n^*}) = \infty$

else evaluate $\Delta P_{n^*}(c_{n^*})$

In the above procedure, if c_{n^*} comes to M , ΔP_{n^*} should be set to the infinite value to prevent more bit loading. Even though the above algorithm is not necessarily optimal, it plays as a good heuristic. The optimality is guaranteed when $k_n = 1$ for all n [9], [17].

V. PROPORTIONAL FAIRNESS

In this section, we develop the optimum IP solution for proportional fairness (PF) by linearizing the objective function in Eq. (13). Additionally, we present the suboptimum scheme separating subcarrier allocation and bit loading. Although the scheduling algorithm is changed into PF one, the bit loading algorithm, the *modified Levin-Campello algorithm*, remains unchanged. For this reason, we focus only on PF subcarrier allocation.

A. Optimum Algorithm - Integer Programming

In Eq. (13), the object function is definitely nonlinear since it includes the product of users' data rate. However, the following theorem converts it into a linear one by simplifying the PF optimization problem.

Theorem 3 (Simplified PF Optimization Problem): As T_W increases, the PF optimization problem in Eq. (13) is asymptotically equivalent to the following one:

$$\begin{aligned} & \max_{c_n, \rho_{k,n}} \sum_{k=1}^K \sum_{n=1}^N \frac{c_n \rho_{k,n}}{R_k(t-1)}, \\ & \text{subject to given } R_k(t-1) \\ & \text{and } \sum_{n=1}^N \max_k \left(\frac{f(c_n) \rho_{k,n}}{\alpha_{k,n}^2} \right) \leq P_T. \end{aligned} \quad (26)$$

Proof: See Appendix III. ■

Especially for $T_W = 1000$ suggested in [16], the simplified optimization problem is almost the same as the original one. Using Theorem 3 and adopting $\gamma_{k,n,c}$, we obtain the IP optimization problem for PF as follows:

$$\begin{aligned} & \max_{\gamma_{k,n,c}} \sum_{k=1}^K \sum_{n=1}^N \sum_{c=1}^M \frac{c \cdot \gamma_{k,n,c}}{R_k(t-1)} \\ & \text{subject to } R_k(t-1) \end{aligned} \quad (27)$$

and the same constraints as those in Eq. (18).

The only difference with the IP problem for MT is that $1/R_k(t-1)$ is additionally applied to the object function to reflect the hysteresis of users' throughput. Even though the above IP problem provides the optimum solution, it still has a critical implementation problem that the computational complexity increases exponentially with the number of constraints and variables. This asks for a suboptimum simple solution.

B. Suboptimum Algorithm - Two Step Approach

Similar to the case of the MT optimization problem, we adopt a two-step approach separating subcarrier allocation and bit loading. A bit loading algorithm simply follows the *modified Levin-Campello algorithm*; hence, we consider only PF subcarrier allocation.

Similar to MT subcarrier allocation, suppose that $P_n = P_T/N$ only for the stage of subcarrier allocation. Then, the optimization problem in Eq. (26) can be rewritten as

$$\begin{aligned} & \max_{c_n, \rho_{k,n}} \sum_{k=1}^K \sum_{n=1}^N \frac{c_n \rho_{k,n}}{R_k(t-1)}, \\ & \text{subject to given } R_k(t-1) \\ & \text{and } \max_k \left(\frac{f(c_n) \rho_{k,n}}{\alpha_{k,n}^2} \right) \leq \frac{P_T}{N}. \end{aligned} \quad (28)$$

With the assumption of equal transmit power, the optimization becomes quite simpler because the user selection for each subcarrier does not affect those for other subcarriers. In other words, subcarrier allocation can be performed in an independent manner. For convenience, we define $\rho_{k,n}^{(i)}$ denoting

the subcarrier indicator when the i th user is selected as the user requiring maximum power, i.e.,

$$\rho_{k,n}^{(i)} = \begin{cases} 1, & \alpha_{k,n}^2 \geq \alpha_{i,n}^2 \\ 0, & \text{otherwise.} \end{cases} \quad (29)$$

Let $c_{k,n}$ be the number of bits that can be received by the k th user through the n th subcarrier in the case of $\rho_{k,n} = 1$, i.e.,

$$c_{k,n} = \min \left(f^{-1} \left(\frac{\alpha_{k,n}^2 P_T}{N} \right), M \right), \quad (30)$$

where $f^{-1}(\cdot)$ is the inverse function of $f(\cdot)$ defined in Eq. (10). Since $f(\cdot)$ is monotonically increasing with c , the inverse function can be uniquely determined.

Using these, the detail procedures are described as follows:

< PF Subcarrier Allocation >

- 1) Evaluate $c_{k,n}$ and $\rho_{k,n}^{(i)}$ for all $i \in \{1, 2, \dots, K\}$, k , and n by using in Eq. (30) and Eq. (29).
- 2) Select the user index maximizing the sum of proportional data rate as follows:

$$\kappa_n = \arg \max_{1 \leq i \leq K} \left[c_{i,n} \cdot \sum_{k=1}^K \frac{\rho_{k,n}^{(i)}}{R_k(t-1)} \right]. \quad (31)$$

- 3) Complete the subcarrier allocation as follows:

$$\rho_{k,n} = \begin{cases} 1, & \alpha_{k,n}^2 \geq \alpha_{\kappa_n,n}^2 \\ 0, & \text{otherwise.} \end{cases} \quad (32)$$

Unlike MT subcarrier allocation, the subcarrier is allocated in a way that maximizes the sum of proportional data rate as shown in Eq. (31). After completing PF subcarrier allocation, bits are loaded according to the *modified Levin-Campello algorithm*.

As mentioned before, the above solution asymptotically approaches the optimum one for large T_W . However, a large value of T_W may increase system *latency* especially when the user's channel condition deteriorates abruptly. In addition, *latency* could be a critical performance measure especially for multimedia services; hence, the performance should be reconsidered in small T_W . In [18], the authors provided an iterative PF allocation scheme in multicarrier systems, which compensates the performance degradation of conventional PF scheduler in the case of small T_W . Inspired by this result, for small T_W , we can employ an iterative approach, where subcarrier allocation is performed for only one subcarrier at a time and then the average data rate $R_k(t-1)$ is updated every allocation to reflect the previous allocations. The detail procedures are described as follows.

< Iterative PF Subcarrier Allocation >

- 1) Evaluate $c_{k,n}$ and $\rho_{k,n}^{(i)}$ for all $i \in \{1, 2, \dots, K\}$, k , and n by using in Eq. (30) and Eq. (29).
- 2) Let $R_k^* = \left(1 - \frac{1}{T_W}\right) R_k(t-1)$, (tentative data rate).
- 3) Let $\mathcal{N} = \{1, 2, \dots, N\}$, (tentative subcarrier set).
- 4) Select the subcarrier and user index maximizing the sum of proportional data rate as follows:

$$\{n^*, \kappa_{n^*}\} = \arg \max_{n \in \mathcal{N}, 1 \leq i \leq K} \left[c_{i,n} \cdot \sum_{k=1}^K \frac{\rho_{k,n}^{(i)}}{R_k^*} \right]. \quad (33)$$

- 5) Perform the subcarrier allocation for selected subcarrier as follows:

$$\rho_{k,n^*} = \begin{cases} 1, & \alpha_{k,n^*}^2 \geq \alpha_{\kappa_{n^*},n^*}^2 \\ 0, & \text{otherwise.} \end{cases} \quad (34)$$

- 6) Update $R_k^* = R_k^* + \frac{1}{T_W} c_{\kappa_{n^*},n^*} \rho_{k,n^*}$ for all k .

- 7) Update $\mathcal{N} = \mathcal{N} - \{n^*\}$.

- 8) If $\mathcal{N} = \emptyset$, complete the procedure, else go to 4).

When compared to non-iterative method, it has two differences. At first, the denominator R_k^* is updated every iteration. Secondly, the subcarrier and user selection are simultaneously performed for the candidate subcarrier set \mathcal{N} , whose cardinal decreases from N to 1, by one every iteration. Hence, the iterative algorithm requires $K \sum_{i=1}^N (N+1-i) = KN(N+1)/2$ comparisons, whereas the non-iterative one requires only KN comparisons. However, although the enhanced technique is applied to cope with small T_W , the performance difference between the non-iterative and iterative algorithms is negligible, which will be numerically shown in Section VI. Therefore, it is sufficient to use non-iterative PF allocation scheme.

VI. NUMERICAL RESULTS

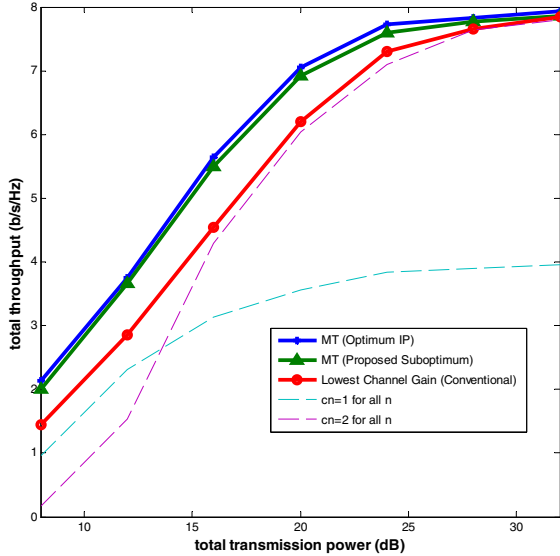
In multicast multicarrier systems, the proposed optimum and suboptimum algorithms for MT and PF are compared with the conventional multicast transmission technique, named lowest channel gain (LCG) method, where all the subcarriers are shared by all the users and bits are loaded using the modified Levin-Campello algorithm. In the case of LCG method, only bit loading information is required at the receiver because all the users share the subcarrier. In order to show the efficacy of the proposed suboptimum algorithms, we compare them with the optimum solutions for small numbers of subcarriers and users.⁵ To test the feasibility of the proposed suboptimum algorithm in practical point of view, they are compared with LCG method for the case of large values of parameters. Finally, by comparing different scheduling algorithms in terms of both throughput and fairness, we show that PF scheduling is most suitable in practical multicast multicarrier systems.

Simulations are performed under the following assumptions. The channel is a frequency selective Rayleigh fading channel with the exponential gain profile. The users are uniformly distributed in a cell, and a large-scale path loss is 2. The average channel gain indicating long-term fading is set to constant, but the short-term fading channel is independently generated every scheduling slot. In fact, 4000 independent short-term fading channels are generated and the results in figures denote the average values over 4000 scheduling slots. The average window size T_W for updating $R_k(t)$ changes from 10 to 1000. The required BER is $p_e = 10^{-4}$ and the noise variance $N_o/2 = 1$. The number of users K is between two to 128.

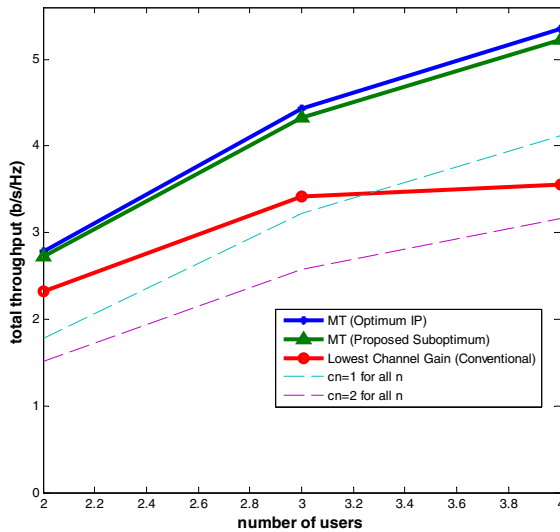
A. Maximum Throughput

Fig. 4 shows the comparison of IP optimum algorithm and suboptimum two-step algorithm when the number of

⁵In simulations, we have difficulty in obtaining the optimum solution for large values of parameters, due to large number of constraints in IP problem.



(a)



(b)

Fig. 4. Maximum Throughput: Comparison of IP optimum algorithm and suboptimum two-step algorithm for $N = 8$ and $M = 2$ (a) variation of total transmission power for $K = 4$ (dB) (b) variation of number of users for $P_T = 16$ dB

subcarriers $N = 8$, maximum loaded bits $M = 2$, and the number of channel taps is four. In Fig. 4(a), the performance difference between the optimum and suboptimum algorithms is within about 5% for a wide range of transmission power, which indicates that the equal power assumption for all subcarriers is reasonable during subcarrier allocation. Compared to the LCG scheme and fixed modulation, e.g., $c_n = 1$ or $c_n = 2$, it can be said that the performance difference between the optimum and suboptimum algorithms is not significant. For large transmission power, we observe that throughput is saturated regardless of any type of algorithms. It is because the maximum loaded bits are limited by two.

In Fig. 4(b), in order to evaluate total data rate for variable

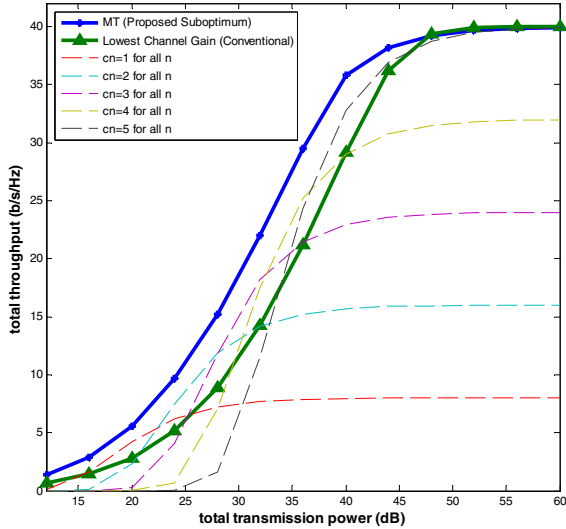
number of users, K is changed from two to four with the transmission power $P_T = 16$ dB. In the case of the optimum/suboptimum algorithms, total throughput increases with the number of users. In the case of LCG method, however, data rate becomes saturated. This is mainly attributed to the fact that in probability the value of the lowest channel gain becomes smaller as the number of users increases. Thus, it should be noted that the proposed scheme can be more useful especially when there are many users. In fact, we have difficulty in obtaining the comparison result in the case of large N , K , and M because the number of constraints in IP problem exponentially increase with N , M , and K . Therefore, the comparison in the case of large parameters were omitted because of simulation problem.

To show the performance gain of the proposed suboptimum algorithm in practical multicarrier (OFDM) systems such as 802.11 [19], we consider the case of large parameters, e.g., $N = 64$ and $M = 5$. In Fig. 5(a), total data rate with varying total transmission power is shown when $K = 8$ and the maximum channel length is eight. For a wide range of transmission power, the proposed algorithm outperforms the LCG scheme. For large transmission power, however, the LCG scheme is slightly better. Nevertheless the proposed suboptimum scheme is meaningful, since we are more interested in the case of insufficient transmission power in practical systems. In Fig. 5(b), we can see that the throughput of LCG method becomes saturated as the number of users increases. On the other hand, the throughput of the proposed algorithm increases with the number of users. It shows the benefit of multicasting services that much more information can be shared with the increase of users. This implies that if an intelligent resource allocation scheme is applied for multicast data, the benefit of multicast system that data are delivered to a group of users can be maximized even in the wireless fading channel. Finally, Fig. 5(c) shows that the throughput is the same as varying the amount of frequency selectivity by increasing the number of taps from one to 16. This implies that throughput is almost independent from channel environment.

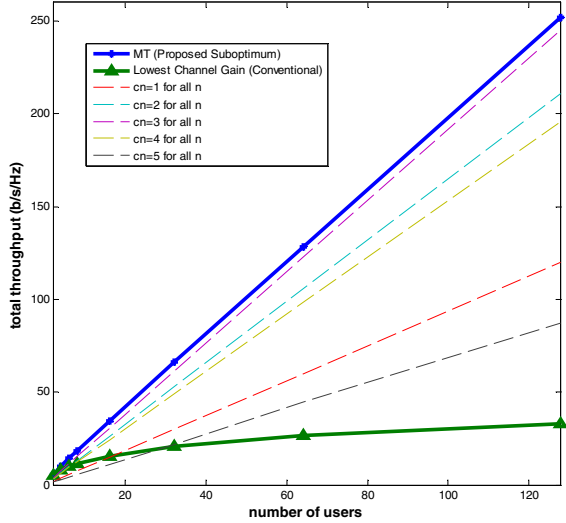
B. Proportional Fairness

Fig. 6 shows the comparison of the IP optimum solution and proposed suboptimum algorithms (two versions) when the number of subcarriers is $N = 4$, maximum loaded bits is $M = 2$, the number of users is four, and the average window size is $T_W = 1000$. As a measurement of proportional fairness, we used the sum of logarithmic data rate. When compared to the LCG method, the performance gap is not significant, which implies that the proposed suboptimum algorithms are well developed. In addition, for $T_W = 1000$, we observe that the iterative algorithm has almost the same performance as the non-iterative one. Although we can see the performance difference for small T_W in Fig 7, it is negligible. This implies that it is sufficient to employ the non-iterative PF allocation in multicarrier multicast systems.

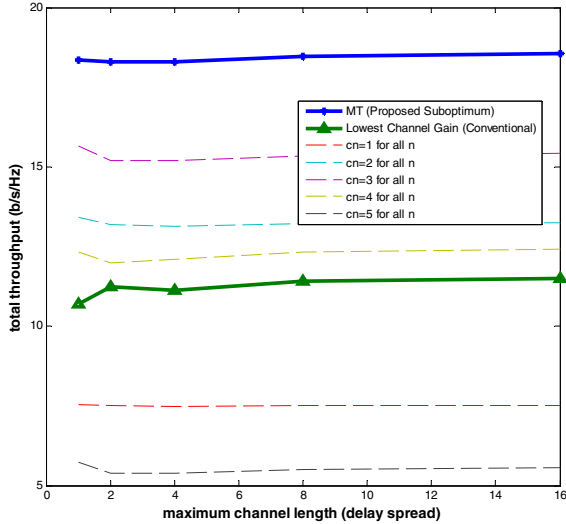
In Fig 8, it is shown that the proposed suboptimum algorithm has performance gain in practical multicarrier systems



(a)



(b)



(c)

Fig. 5. The performance of maximum throughput (MT) scheduler for $N = 64$ and $M = 5$ (a) variation of total transmission power (dB) for $K = 8$ (b) variation of number of users for $P_T = 30$ dB (c) variation of maximum channel length for $K = 8$ and $P_T = 30$ dB

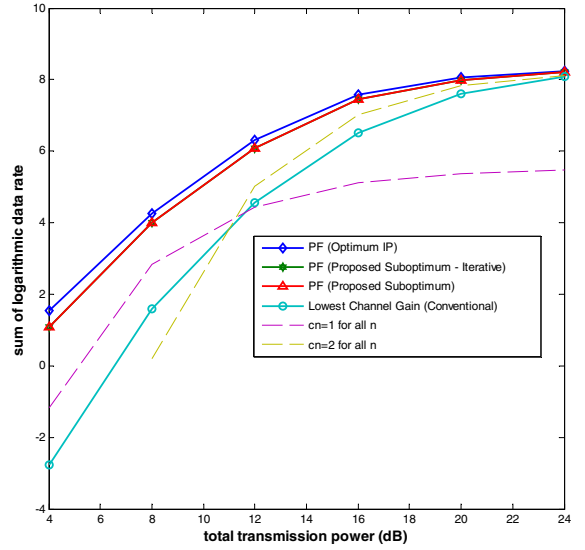


Fig. 6. Proportional Fairness: comparison of the optimum IP solution and the proposed suboptimum algorithms for $N = 4$, $K = 4$, $M = 2$, and $T_W = 1000$

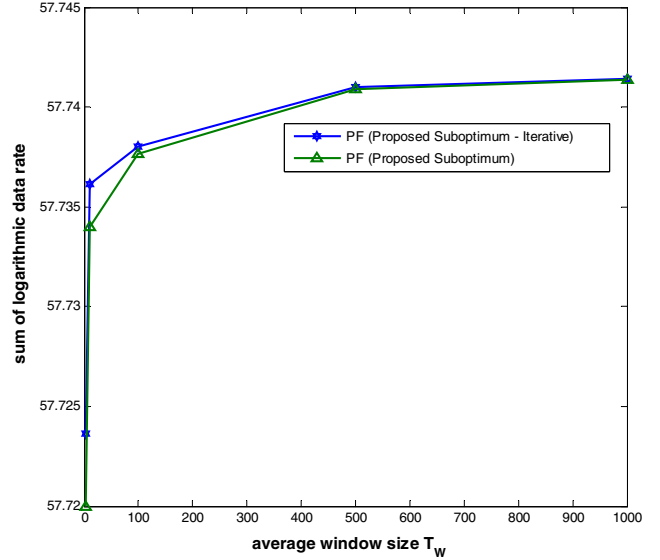
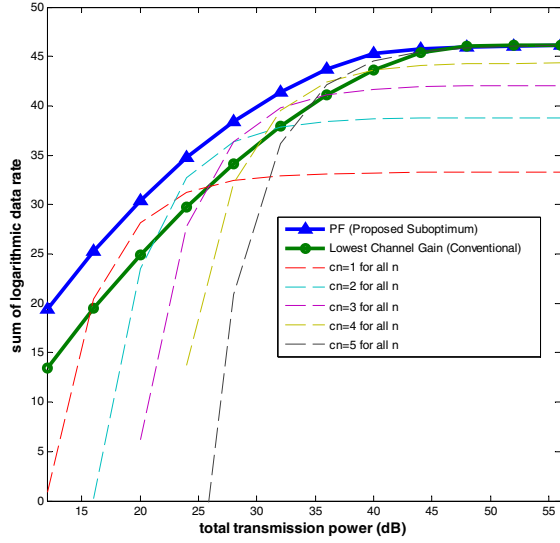


Fig. 7. Proportional Fairness: performance difference between non-iterative and iterative allocation schemes for $N = 64$, $K = 8$, $M = 5$, and $P_T = 30$ dB

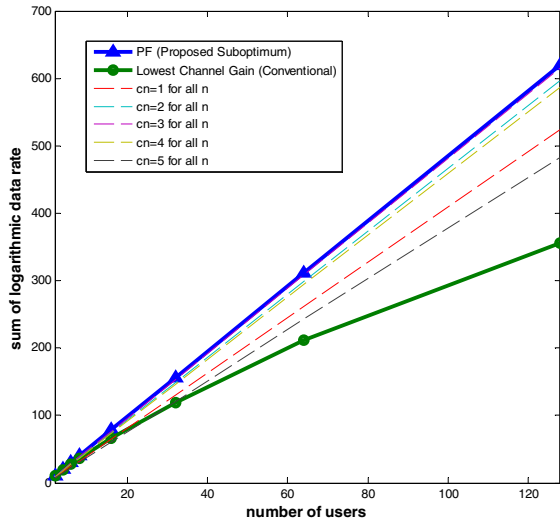
adopting large N . In Fig. 8(b), we can observe that the performance gap between the proposed algorithm and LCG method becomes larger as the number of users increases. It means that the proposed algorithms for PF scheduling becomes more important as the number of users increases. In Fig. 8(c), we observe that the performance of the proposed PF algorithm is independent from channel environment.

C. Comparison of Different Scheduling Algorithms

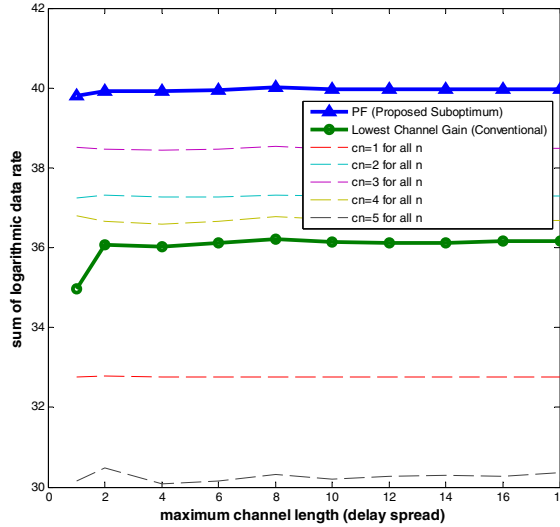
Fig. 9(a) shows the throughput comparison of three scheduling algorithms, i.e., maximum throughput (MT), proportional



(a)

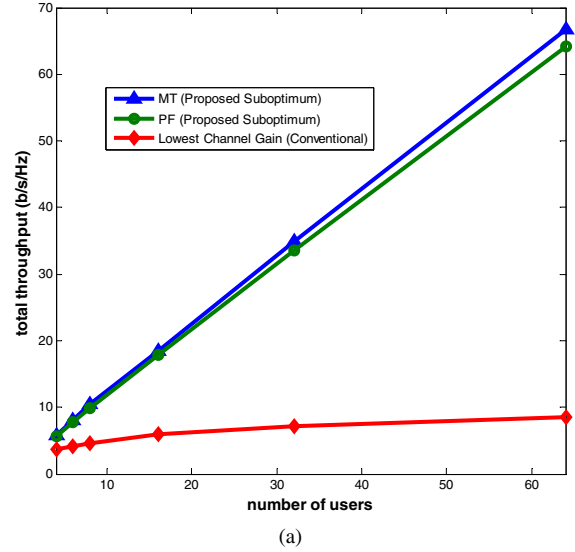


(b)

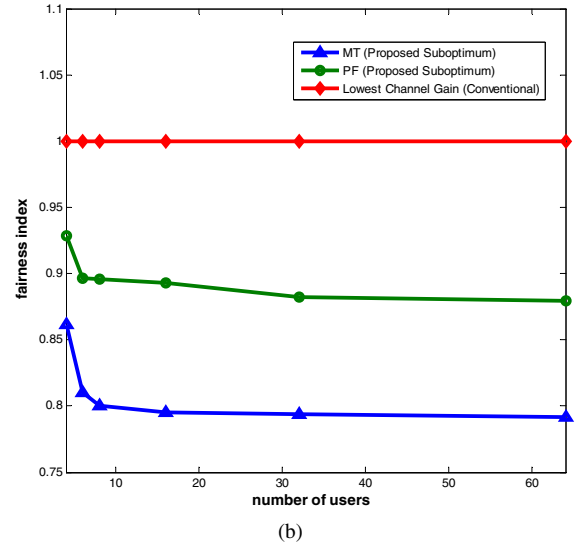


(c)

Fig. 8. The performance of proportional fairness (PF) scheduler for $N = 64$ and $M = 5$ (a) variation of total transmission power (dB) for $K = 8$ (b) variation of number of users for $P_T = 30$ dB (c) variation of maximum channel length for $K = 8$ and $P_T = 30$ dB



(a)



(b)

Fig. 9. Throughput-Fairness comparison of different scheduling algorithms, i.e., MT, PF, and LCG, for $N = 64$, $M = 5$, $T_W = 1000$, and $P_T = 30$ dB (a) throughput performance (b) fairness performance

fairness (PF), and lowest channel gain (LCG) when the number of subcarriers is 64 and the number of maximum loaded bits is five. In this figure, the performance gap between MT and PF schedulers is small when compared to the LCG method. In addition, although the throughput difference between MT and PF becomes large with the increase of number of users, it is still within about 5% for a wide range of number of users.

In order to evaluate the fairness of each scheduling scheme, Fig. 9(b) shows the fairness performance as a function of number of users. As a measurement of fairness, we adopted the following fairness index (FI) defined in [20]:

$$FI = \frac{(\sum_k R_k(t))^2}{K (\sum_k R_k^2(t))}. \quad (35)$$

As shown in Fig. 9(b), from the perspective of fairness, LCG method has the best performance among three scheduling

schemes as expected. The PF scheduling has compromised performance between MT and LCG method.

Based on Fig. 9, we can say that even for multicast services PF scheduling is a compromised technique to guarantee the fairness while minimizing throughput degradation. Thus PF scheduling may be the best solution for practical multicast multicarrier systems.

VII. DISCUSSIONS

In this paper, we would like to mention the positive and negative nature of multicast services in wireless channels. An attractive merit of multicasting in wireless channel is that multicast data can be delivered to many users only through a *single* transmission without increasing any wired connections. However, the problem is the difference of link conditions of users, which limits the multicast capacity. As a way of overcoming link asymmetry, the user grouping method can be easily expected where the users having similar channel conditions are grouped for multicast transmission. However, the problem is that another bandwidth is required to support other users excluded from user grouping. Multicarrier systems alleviate this problem by providing additional bandwidth for those users. In addition, hierarchical data structure can be efficiently exploited for user grouping, since it determines the data quality according to the amount of received data.

Accordingly, the lesson from the above observations is that multicast services can be efficiently provided when *hierarchical data* structure is exploited for *multicarrier wireless* systems. We would like to emphasize that the proposed resource allocation schemes are the very results acquired from well understanding of this lesson.

As a tradeoff, the proposed algorithms also have a disadvantage of increasing downlink control overhead due to subcarrier/bit allocation information. It is an unavoidable cost. However, the cost may not be high especially for fixed indoor environment with low mobility. In addition, this paper still needs to validate the assumption that any combination of layers consisting of multicast data can be decoded at the receiver. This asks for an intelligent mapping algorithm for efficiently recovering the original data from different layers. It could be a further work.

VIII. CONCLUSIONS

Using the integer programming, the optimum resource allocation algorithm for MT and PF scheduling have been developed for the efficient usage of scarce spectrum in multicast multicarrier systems under the assumption that hierarchical data can be combined using the received fragmented data. To reduce the complexity of IP algorithm, we also proposed the suboptimum algorithms for MT and PF, separating subcarrier allocation and bit loading. Through the simulations, it was shown that the proposed algorithms significantly outperform the conventional LCG method and performance difference between the optimum and the proposed suboptimum algorithms was within about 5% for both MT and PF. In addition, we have shown that PF scheduling is the most suitable technique

for multicast multicarrier systems considering both throughput and fairness.

APPENDIX I PROOF OF LEMMA 1

Using the pdf of the first order statistics:

$$f_{(1)}(y) = \frac{K}{\alpha_{(1)}^2} e^{-\frac{K}{\alpha_{(1)}^2} y}, \quad (36)$$

we have

$$\begin{aligned} E \left[K \cdot \log_2 \left(1 + \frac{P}{\sigma^2} Y_1 \right) \right] \\ = K \cdot \int_0^\infty \log_2 \left(1 + \frac{P}{\sigma^2} y \right) f_{(1)}(y) dy \\ = \frac{K^2}{\alpha_{(1)}^2} \int_0^\infty \log_2 \left(1 + \frac{P}{\sigma^2} y \right) e^{-\frac{K}{\alpha_{(1)}^2} y} dy. \end{aligned} \quad (37)$$

In page 568 in [8], there is the following integral equation with regard to the exponential and log functions:

$$\int_0^\infty e^{-ax} \ln(1+bx) dx = -\frac{1}{a} e^{\frac{a}{b}} \text{Ei} \left(-\frac{a}{b} \right), \quad (38)$$

for $|\angle(b)| < \pi$ and $\Re(a) > 0$,

where $\text{Ei}(-x)$ is an exponential integral function already defined in Lemma 1. Applying Eq. (38) into Eq. (37), we obtain Eq. (4).

APPENDIX II PROOF OF THEOREM 1

For simplicity, if we let $a = \frac{\alpha_{(1)}^2 P}{\sigma^2}$, the expected multicast capacity is rewritten as

$$E[C_{MC}] = -\log_2 e \cdot K e^{\frac{K}{a}} \text{Ei} \left(-\frac{K}{a} \right), \quad (39)$$

Using the exponential integral function defined in Lemma 1, we obtain

$$\begin{aligned} E[C_{MC}] &= \log_2 e \cdot K e^{\frac{K}{a}} \int_{\frac{K}{a}}^\infty \frac{e^{-t}}{t} dt \\ &= \log_2 e \cdot a \int_0^\infty \frac{K e^{-x}}{K + ax} dx \\ &= \log_2 e \cdot a \int_0^\infty \frac{e^{-x}}{1 + \frac{ax}{K}} dx. \end{aligned} \quad (40)$$

For $K = \infty$, we have

$$\begin{aligned} \lim_{K \rightarrow \infty} E[C_{MC}] &= \log_2 e \cdot a \lim_{K \rightarrow \infty} \int_0^\infty \frac{e^{-x}}{1 + \frac{ax}{K}} dx \\ &= \log_2 e \cdot a \int_0^\infty e^{-x} dx \\ &= \log_2 e \cdot a = \log_2 e \cdot \frac{\alpha_{(1)}^2 P}{\sigma^2}. \end{aligned} \quad (41)$$

Therefore, it completes the proof of Theorem 1.

APPENDIX III
PROOF OF THEOREM 2

Using Eq. (12), the object function in the real PF optimization problem can be rewritten as

$$\begin{aligned}
 \prod_{k=1}^K R_k(t) &= \left[\left(1 - \frac{1}{T_W} \right)^K \prod_{k=1}^K R_k(t-1) \right] \\
 &\quad \times \prod_{k=1}^K \left(1 + \frac{\sum_n c_n \rho_{k,n}}{(T_W - 1)R_k(t-1)} \right) \\
 &= c_1 \left[1 + \frac{1}{T_W - 1} \sum_{k=1}^K \sum_{n=1}^N \frac{c_n \rho_{k,n}}{R_k(t-1)} + \right. \\
 &\quad \left. \left(\frac{1}{T_W - 1} \right)^2 \sum_{i \neq j} \frac{\sum_n c_n \rho_{i,n} \sum_m c_m \rho_{j,m}}{R_i(t-1)R_j(t-1)} + \dots \right]. \tag{42}
 \end{aligned}$$

Note that c_1 is constant and the higher order terms (≥ 2) can be disregarded for large T_W . Therefore, as T_W increases, the real PF optimization problem becomes asymptotically equivalent to the simplified one defined in Eq. (26). It completes the proof of Theorem 3.

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