Recursive Construction of Golay Sequences

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I. Introduction

Golay complementary sequence (GCS) has been applied to orthogonal frequency division multiplexing (OFDM) systems because it serves not only to generate the training sequence with low PAPR, but also to design the channel code providing both error correction capability and low PAPR [1, 2]. Since the exhaustive search of GCS with a long length is quite difficult, the construction rule of GCS is usually required.

Based on the relation between Reed-Muller codes and GCS, the construction rules of GCS were developed for 2^b-PSK [1] and 16-QAM [2], but the length was restricted to 2^m. Since the different length ($\neq 2^m$) may be required in practical OFDM systems with null subcarriers in DC and high frequencies, the above constructions may not be used. In [3], it was proved that GCS certainly exists at the length of $2^\alpha 10^\beta 26^\gamma$[1], but the recursive construction has been provided for only BPSK.

In this paper, in order to give a flexibility to GCS length and the constellation, we present the recursive constructions for QPSK-GCS, 8PSK-GCS, and 8QAM-GCS having the length of $2^\alpha 10^\beta 26^\gamma$. The proposed sequences have the upper bound (3 dB) of PAPR and the constellation aggregation of 8-PSK and 8-QAM constitutes 16-QAM.

II. Recursive Construction

If the polynomial form is employed to express GCS, it helps to think of the recursive construction intuitively. Let $A = (a_0, a_1, \cdots, a_{N-1})$ and $B = (b_0, b_1, \cdots, b_{N-1})$ be GCSs of length N. The polynomial form is defined by $A(x) := a_0 + a_1 x + \cdots + a_{N-1}x^{N-1}$. By the definition of GCS, it can be easily verified that $A(x)A^*(x^{-1}) + B(x)B^*(x^{-1})$ becomes constant, where $A^*(x^{-1}) := a_0^* + a_1^* x^{-1} + \cdots + a_{N-1}^* x^{-N+1}$. This interesting property can motivate the following Theorem, which is the same construction as that mentioned in [4].

Theorem 1 *If $[A(s), B(s)]$ and $[C(r), D(r)]$ are Golay Complementary Sequences (GCSs), the combination pair*

$$\begin{aligned} E(r,s,t,u) &= C(r)A(s) + D^*(r^{-1})B(s)t, \\ F(r,s,t,u) &= (D(r)A(s) - C^*(r^{-1})B(s)t)u \end{aligned} \tag{1}$$

is GCS.

Using the definition of GCS, the above theorem can be easily proved. Note that the recursive constructions developed by Golay [5] and Turyn [3] are the special cases of Theorem 1.

If we exploit Theorem 1 further, QPSK-GCS of length MN can be constructed from two BPSK-GCS pairs of length M and N, respectively. Since BPSK-GCS can be generated from [5] or [3], we can construct QPSK-GCS of length $2^\alpha 10^\beta 26^\gamma$ through the following Corollary.

[1]The GCS may exists for the other length; so to speak, the length criterion has yet to be completely found.

Corollary 1 *If $[C(x), D(x)]$ and $[A(x), B(x)]$ are GCSs of length M and N, respectively, the combination pair*

$$\begin{aligned} E(x) =& C(x^N)(1+\jmath)(A(x)+B(x))/2 \\ &+ D^*(x^{-N})x^{N(M-1)}(1-\jmath)(A(x)-B(x))/2 \\ F(x) =& D(x^N)(1+\jmath)(A(x)+B(x))/2 \\ &- C^*(x^{-N})x^{N(M-1)}(1-\jmath)(A(x)-B(x))/2 \end{aligned} \tag{2}$$

is GCS of length MN.

Starting with BPSK-GCS and QPSK-GCS generated through Corollary 1, 8PSK- or 8QAM-GCS can be constructed as follows:

Corollary 2 *If $[C(x), D(x)]$ and $[A(x), B(x)]$ are BPSK-GCS and QPSK-GCS of length M and N, respectively, the combination pair*

$$\begin{aligned} E(x) =& C(x^N)(3A(x)+B(x))/2 \\ &+ D^*(x^{-N})x^{N(M-1)}(A(x)-3B(x))/2 \\ F(x) =& D(x^N)(3A(x)+B(x))/2 \\ &- C^*(x^{-N})x^{N(M-1)}(A(x)-3B(x))/2 \end{aligned} \tag{3}$$

is 8PSK- or 8QAM-GCS of length MN. For example, if we let $C(x^N) = D(x^N) = 1$, we obtain $[E(x), F(x)] = [2A(x) - B(x), A(x) + 2B(x)]$, which is the same construction used in [2].

At the first glance, Corollary 2 seems to construct 16-QAM Golay sequences but the resultant sequences are restricted to 8-PSK or 8-QAM because QPSK Golay pair generated by Corollary 1 keep the particular relations. In one case, $a_k = b_k$ or $a_k = -b_k$ where the resultant sequences belong to 8-QAM. In the other case, $a_k = b_k^*$ or $a_k = -b_k^*$ where 8-PSK sequences are constructed. The upper bound of PAPR of the proposed sequences is 3 dB since they satisfy the GCS definition.

References

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